A theoretical comparison of different proton imaging set-ups

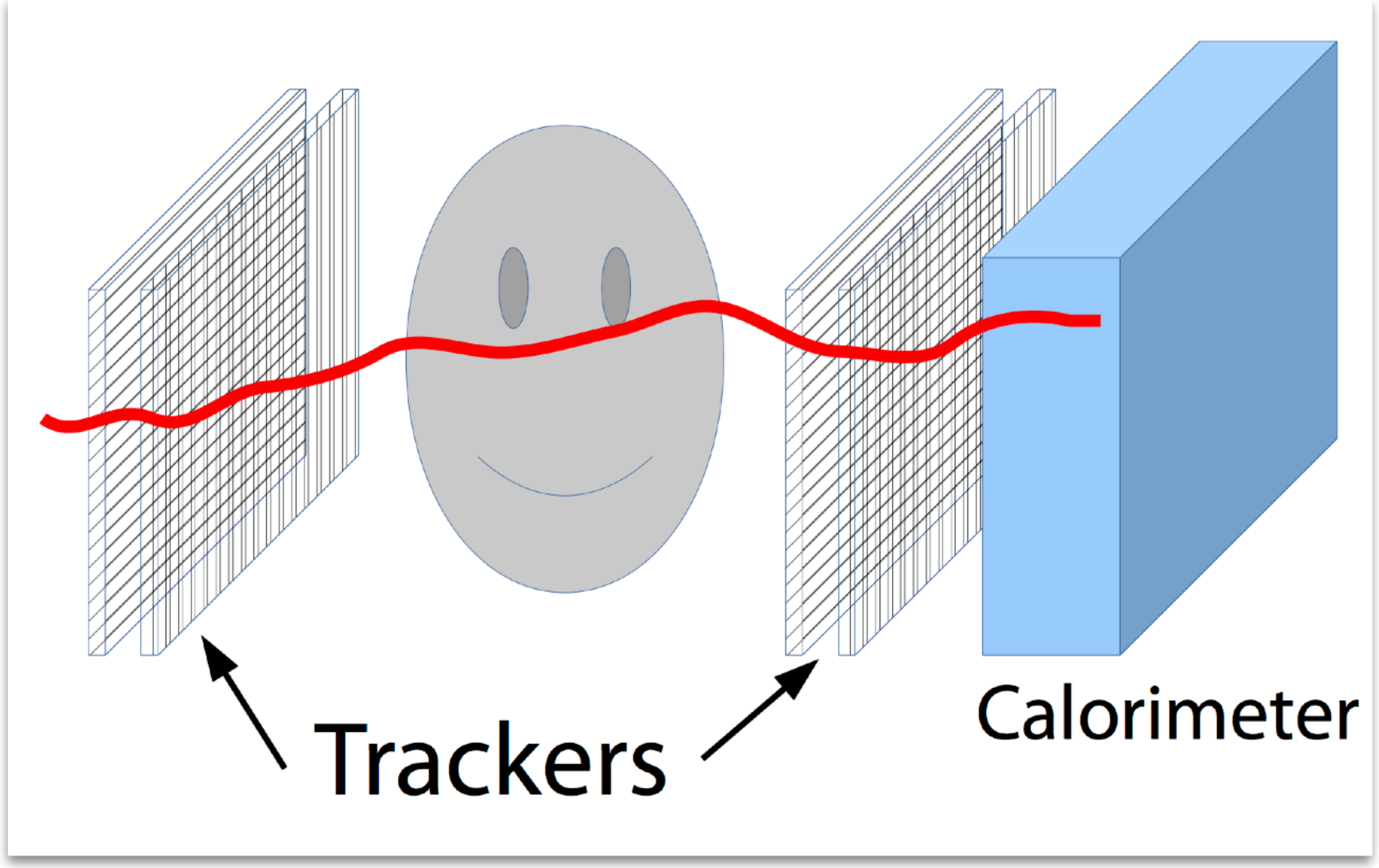
N. Krah[1]

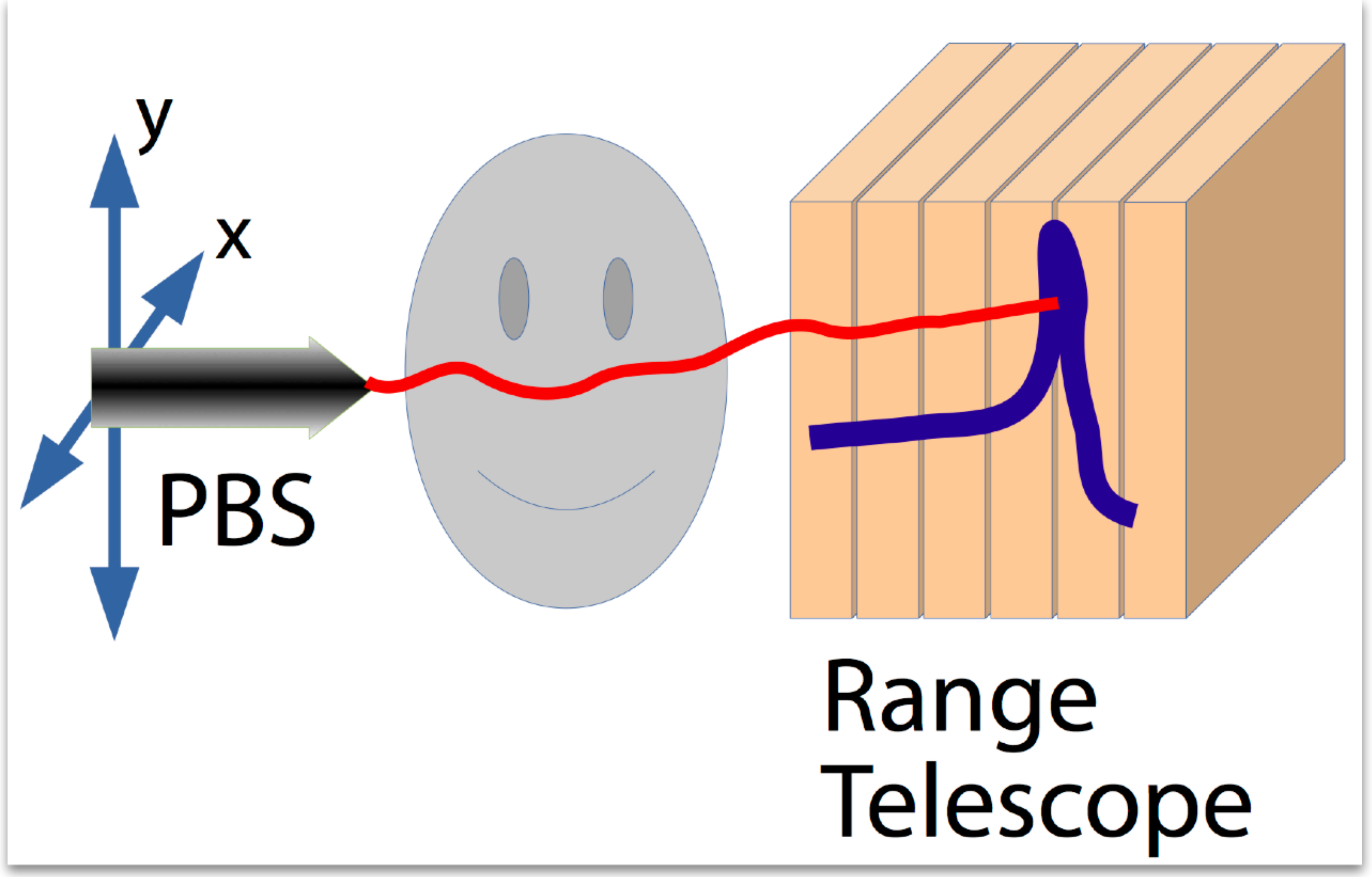
F. Khellaf[1], JM. Létang[2], S. Rit[1], I. Rinaldi[3,4]

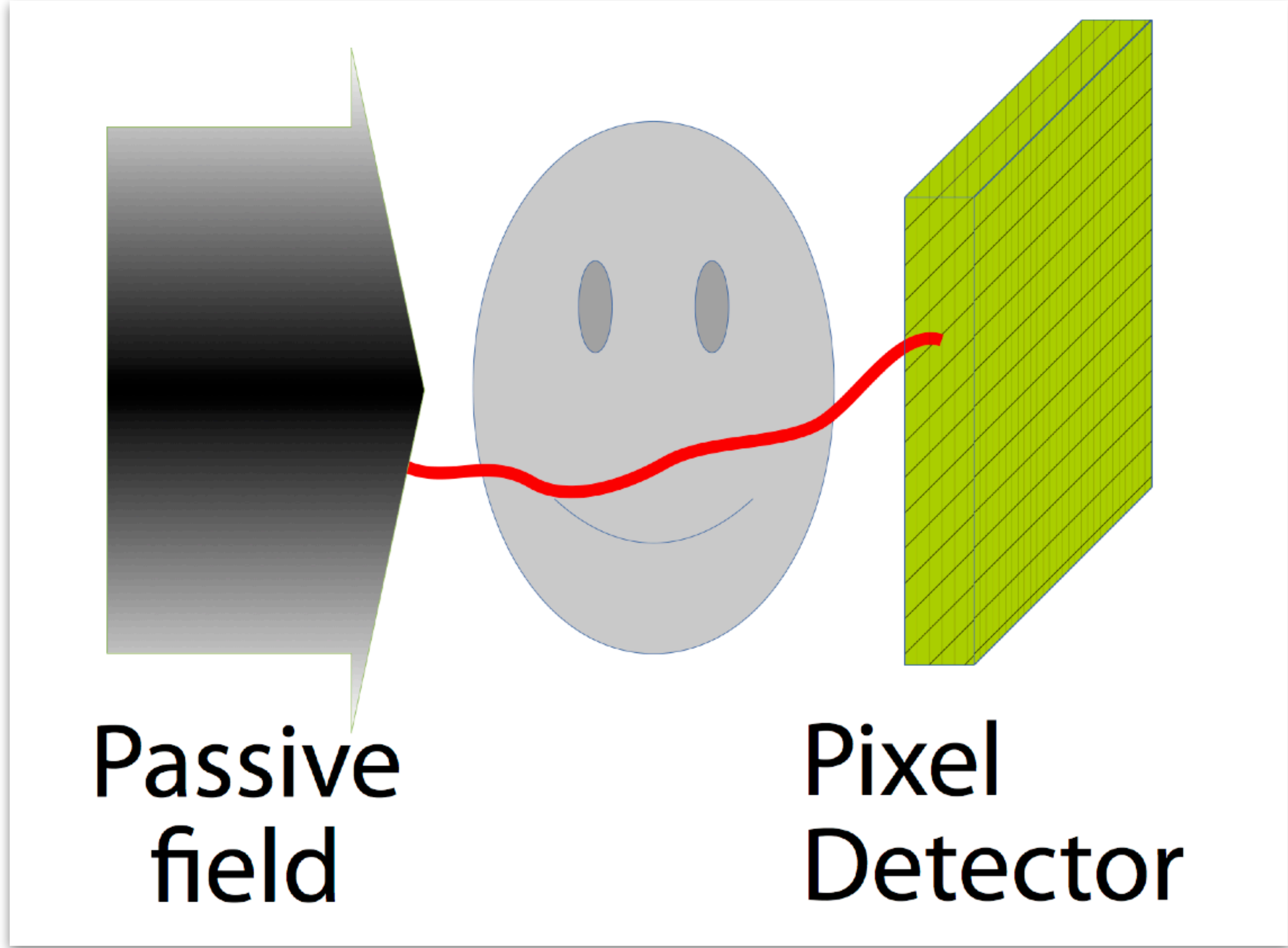
[1]CNRS - Creatis, Lyon, France
[2]INSA Lyon - Creatis, Lyon, France
[3]Maastro Clinic, Maastricht, Netherlands
[4]CNRS - IPNL, Lyon, France

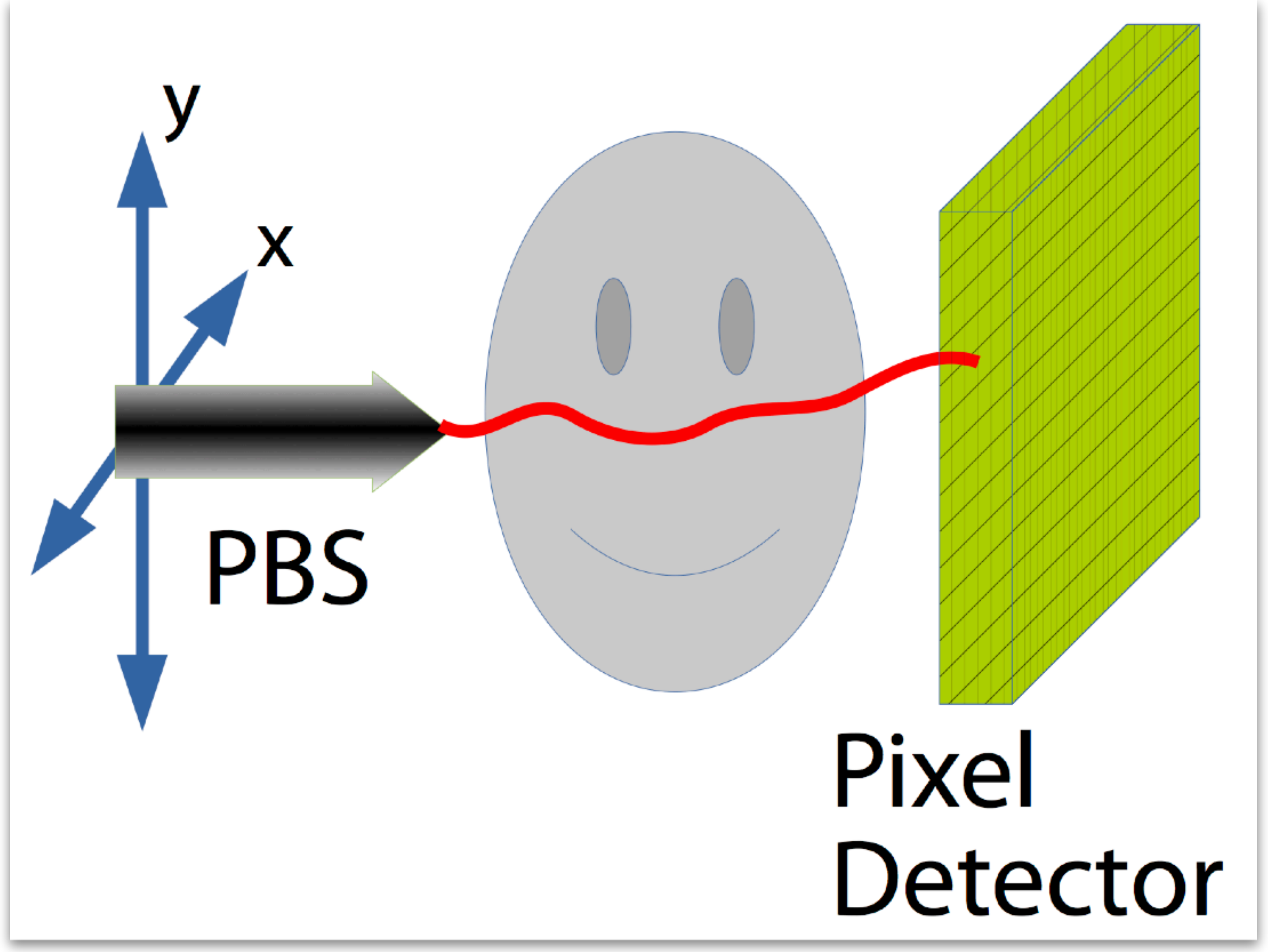
4th LLU Algorithm Workshop

Selection of proton imaging set-ups









Also:

- **Use Helium or carbon instead of protons (-> Lenny)**
- **Pencil beam plus down stream tracker (-> Helge)**

How to choose the set-up?

Cost

Integration into clinical reality

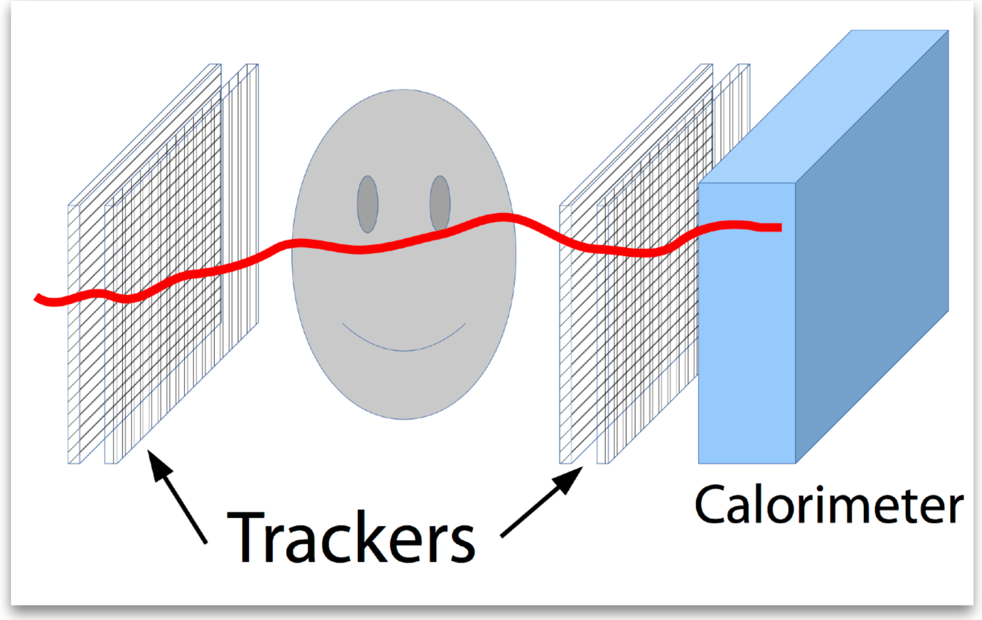
RSP/WET accuracy

Dose to the patient

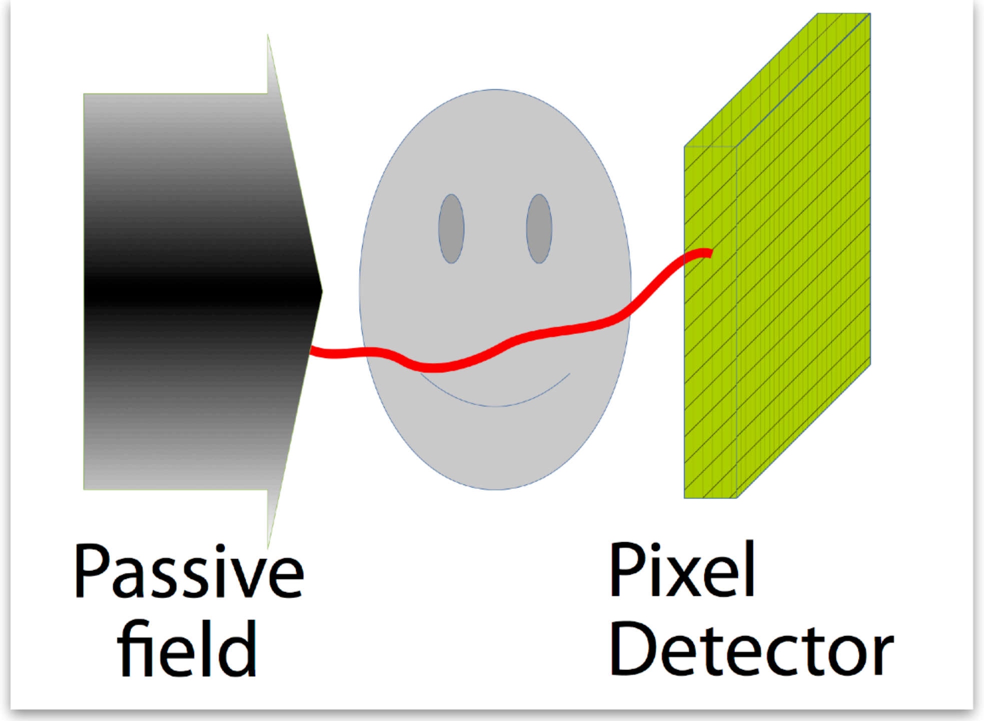
Spatial resolution

Cost

Integration

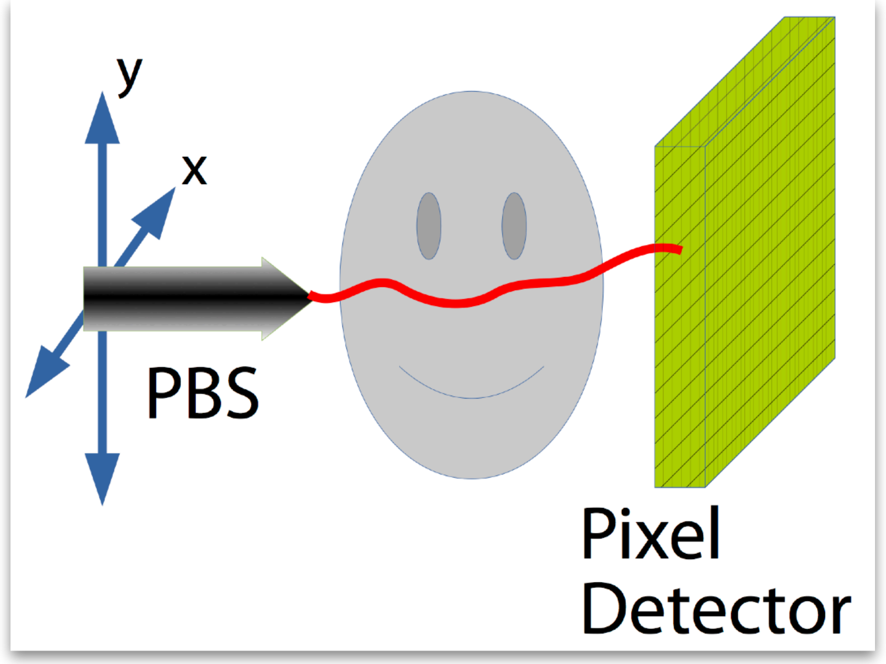


You need to construct:
- **8 tracker devices**
- **1 calorimeter**
- **1 fast electronics**

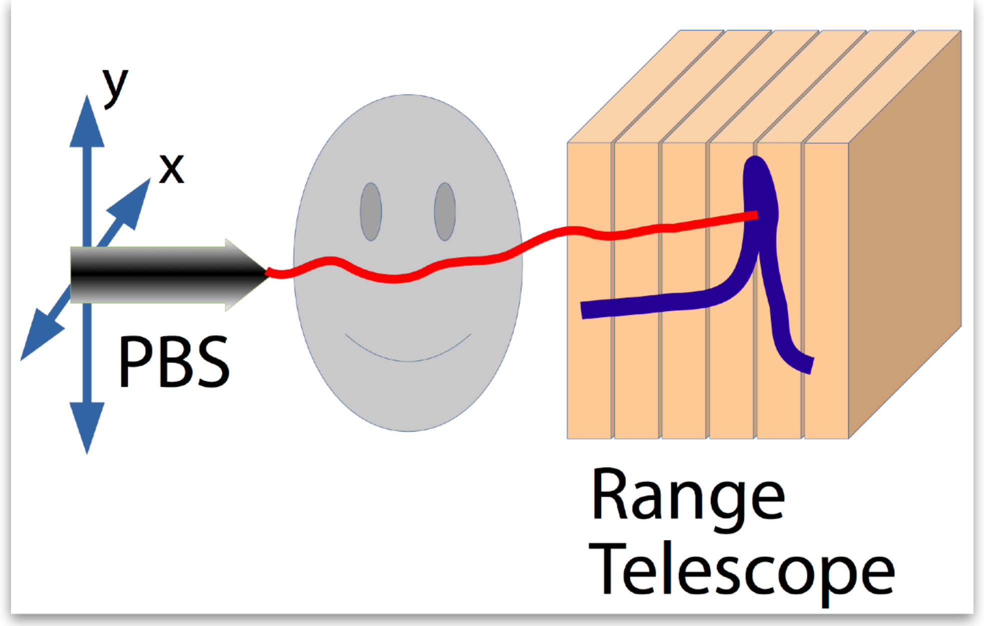


You need to construct:
- **1 Range modulator wheel**

You can re-use:
- **1 X-ray flat panel detector**



You can re-use:
- **1 X-ray flat panel detector**

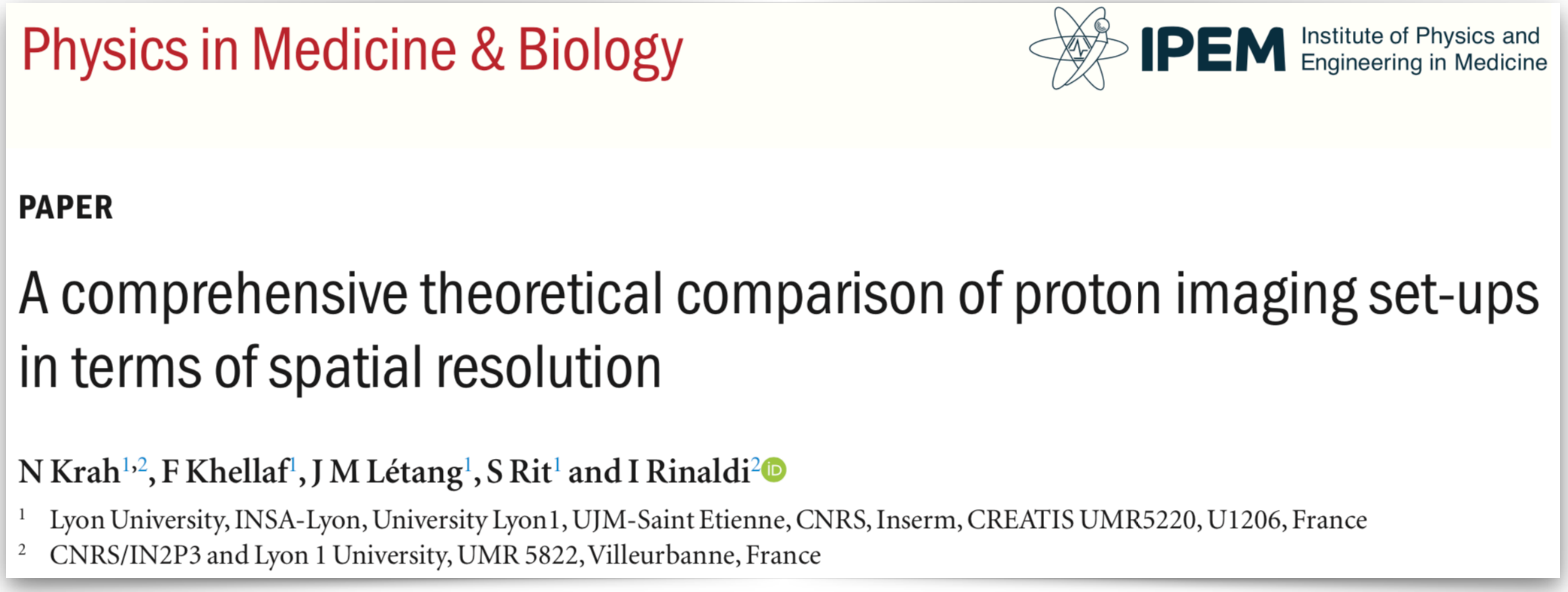


You can re-use:
- **1 Range telescope**

Simple integration into treatment room

Integration into workflow?

Spatial resolution


Physics in Medicine & Biology

IPEM Institute of Physics and Engineering in Medicine

PAPER

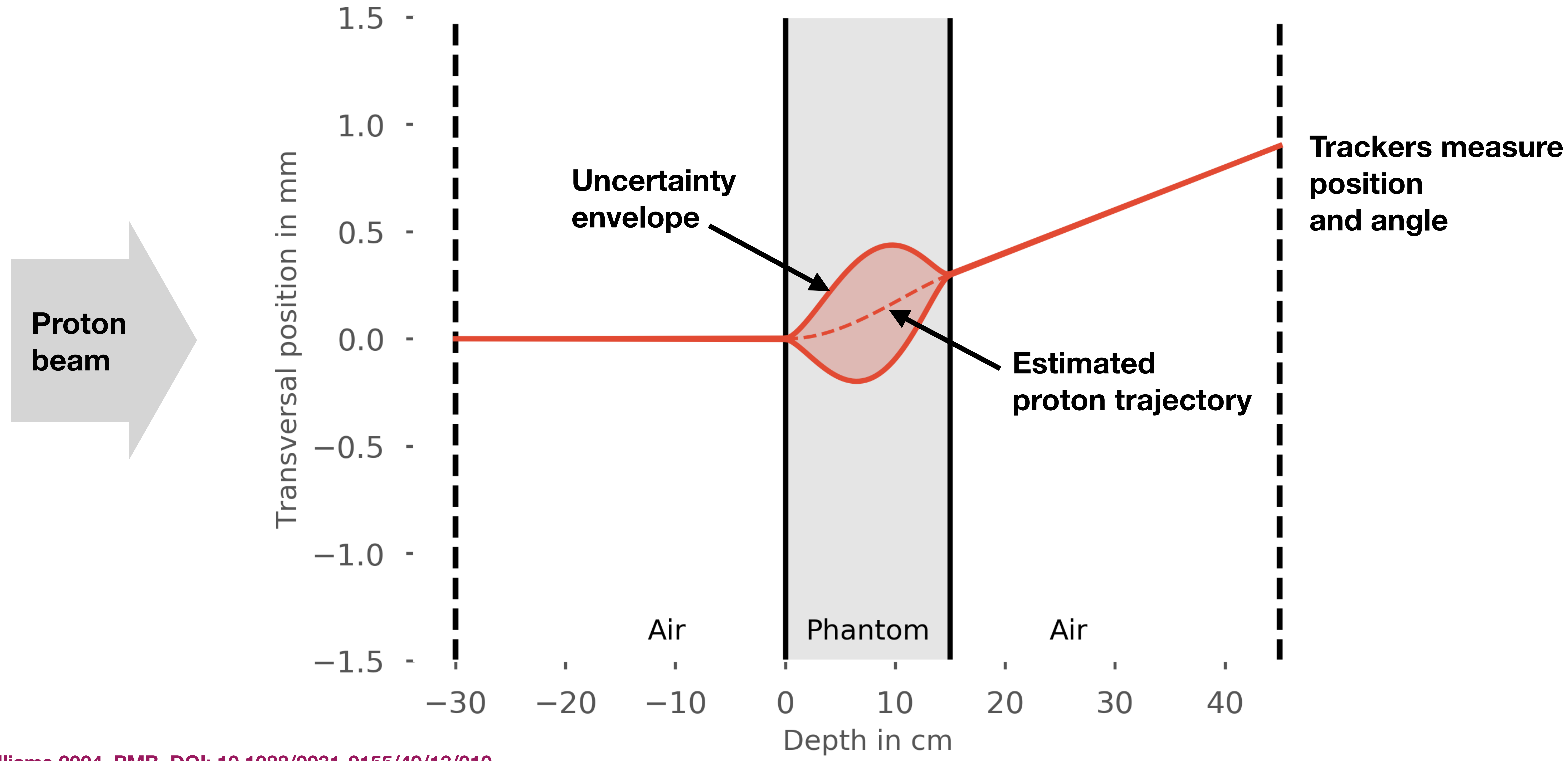
A comprehensive theoretical comparison of proton imaging set-ups in terms of spatial resolution

N Krah[1,2], F Khellaf[1], J M Létang[1], S Rit[1] and I Rinaldi[2]

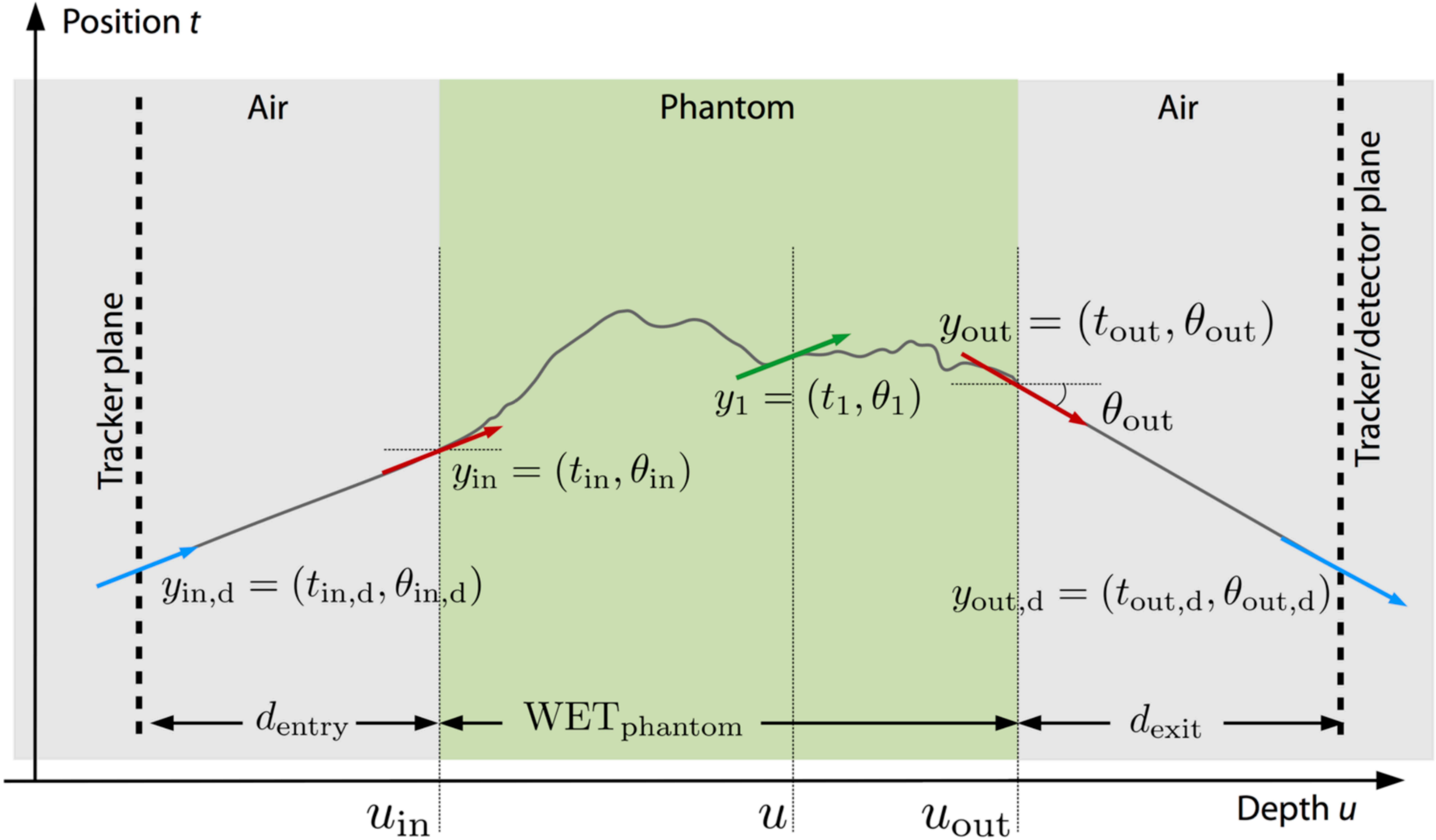
[1] Lyon University, INSA-Lyon, University Lyon1, UJM-Saint Etienne, CNRS, Inserm, CREATIS UMR5220, U1206, France

[2] CNRS/IN2P3 and Lyon 1 University, UMR 5822, Villeurbanne, France

Single tracking set-ups



Williams 2004, PMB, DOI: 10.1088/0031-9155/49/13/010
Schulte 2008, Med. Phys., DOI: 10.1118/1.2986139
Collins-Fekete 2017, PMB, DOI: 10.1088/1361-6560/aa58ce



Position t
Air
Phantom
Air
Tracker plane
Tracker/detector plane
$y_{out} = (t_{out}, \theta_{out})$
$y_1 = (t_1, \theta_1)$
θ_{out}
$y_{in} = (t_{in}, \theta_{in})$
$y_{in,d} = (t_{in,d}, \theta_{in,d})$
$y_{out,d} = (t_{out,d}, \theta_{out,d})$
d_{entry}
$\mathrm{WET}_{phantom}$
d_{exit}
u_{in}
u
u_{out}
Depth u

Likelihood: $$L(y_1, y_2 = y_{\text{out}}|y_{\text{in}}) = L_{\text{scat}}(y_{\text{in}} \to y_1) \times L_{\text{scat}}(y_1 \to y_2 = y_{\text{out}}|y_{\text{in}})$$

from entrance to some depth: $$L_{\text{scat}}(y_{\text{in}} \to y_1) \propto \exp\left[-\frac{1}{2}(y_1^T - y_{\text{in}}^T R_0^T)\Sigma_1^{-1}(y_1 - R_0 y_{\text{in}})\right]$$

from some depth to exit: $$L_{\text{scat}}(y_1 \to y_2 = y_{\text{out}}) \propto \exp\left[-\frac{1}{2}(y_{\text{out}}^T - y_1^T R_1^T)\Sigma_2^{-1}(y_{\text{out}} - R_1 y_1)\right],$$

with: $$\Sigma_1 = \begin{pmatrix} \sigma_{t_1}^2 & \sigma_{t_1\theta_1}^2 \\ \sigma_{t_1\theta_1}^2 & \sigma_{\theta_1}^2 \end{pmatrix}, \Sigma_2 = \begin{pmatrix} \sigma_{t_2}^2 & \sigma_{t_2\theta_2}^2 \\ \sigma_{t_2\theta_2}^2 & \sigma_{\theta_2}^2 \end{pmatrix}, R_0 = \begin{pmatrix} 1 & u - u_{\text{in}} \\ 0 & 1 \end{pmatrix}, R_1 = \begin{pmatrix} 1 & u_{\text{out}} - u \\ 0 & 1 \end{pmatrix}$$

$$\sigma_{t_1}^2 = E_0^2\left(1 + 0.038 \ln \frac{u - u_{\text{in}}}{X_0}\right)^2 \times \int_{u_{\text{in}}}^{u} \frac{(u - u_{\text{in}})^2}{\beta^2 p^2} \frac{\mathrm{d}u}{X_0}$$

Likelihood: $L(y_1, y_2 = y_{\text{out}} | y_{\text{in}}) = L_{\text{scat}}(y_{\text{in}} \to y_1) \times L_{\text{scat}}(y_1 \to y_2 = y_{\text{out}} | y_{\text{in}})$

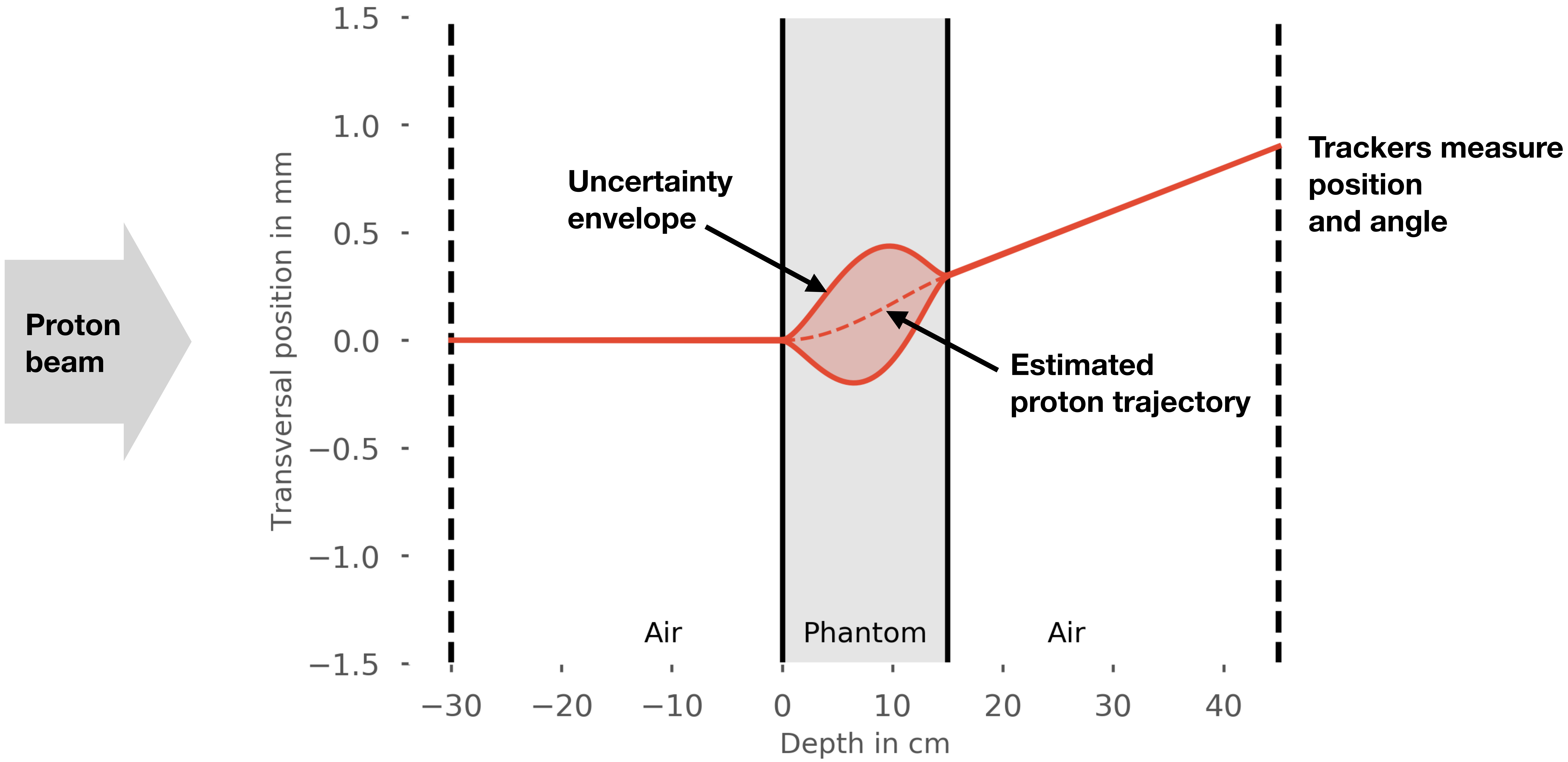
Most likely path (MLP):

$$
\begin{aligned}
y_{\text{MLP}}(u) &= \left(\Sigma_1^{-1} + R_1^T \Sigma_2^{-1} R_1\right)^{-1} \cdot \left(\Sigma_1^{-1} R_0 y_{\text{in}} + R_1^T \Sigma_2^{-1} y_{\text{out}}\right) \\
&= R_1^{-1} \Sigma_2 \left(R_1^{-1} \Sigma_2 + \Sigma_1 R_1^T\right)^{-1} \cdot R_0 y_{\text{in}} + \Sigma_1 \left(R_1 \Sigma_1 + \Sigma_2 (R_1^{-1})^T\right)^{-1} \cdot y_{\text{out}}
\end{aligned}
$$

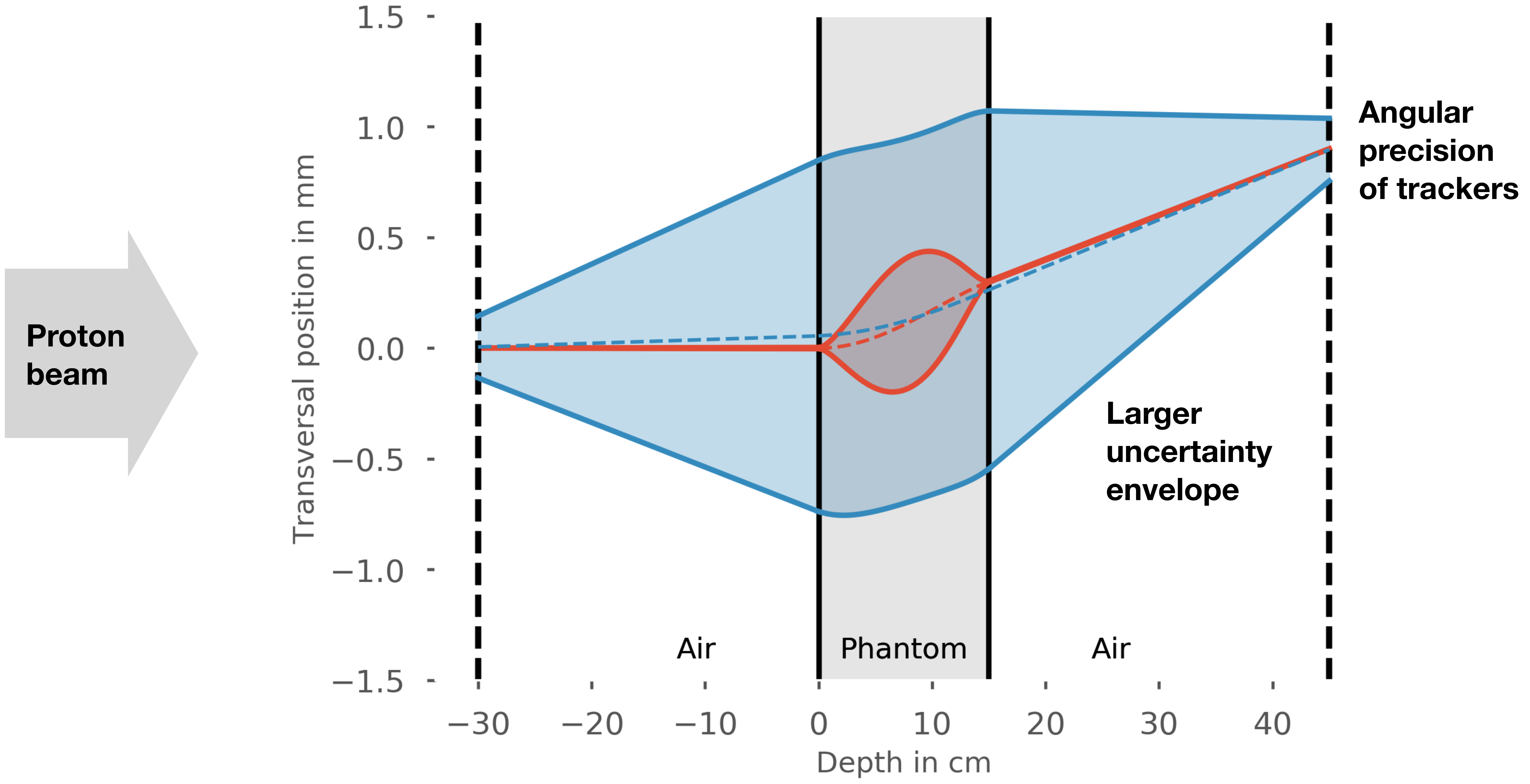
Uncertainty envelope around MLP:

$$
\Sigma_{\text{MLP}}(u) = \left(\Sigma_1^{-1} + R_1^T \Sigma_2^{-1} R_1\right)^{-1} = \Sigma_1 \left(\Sigma_2 (R_1^{-1})^T + R_1 \Sigma_1\right)^{-1} \Sigma_2 (R_1^{-1})^T
$$

Single tracking set-ups



Tracker uncertainties



Bopp et al. 2014, PMB, DOI: 10.1088/0031-9155/59/23/N197
Penfold S 2011, Radiat. Meas. 46

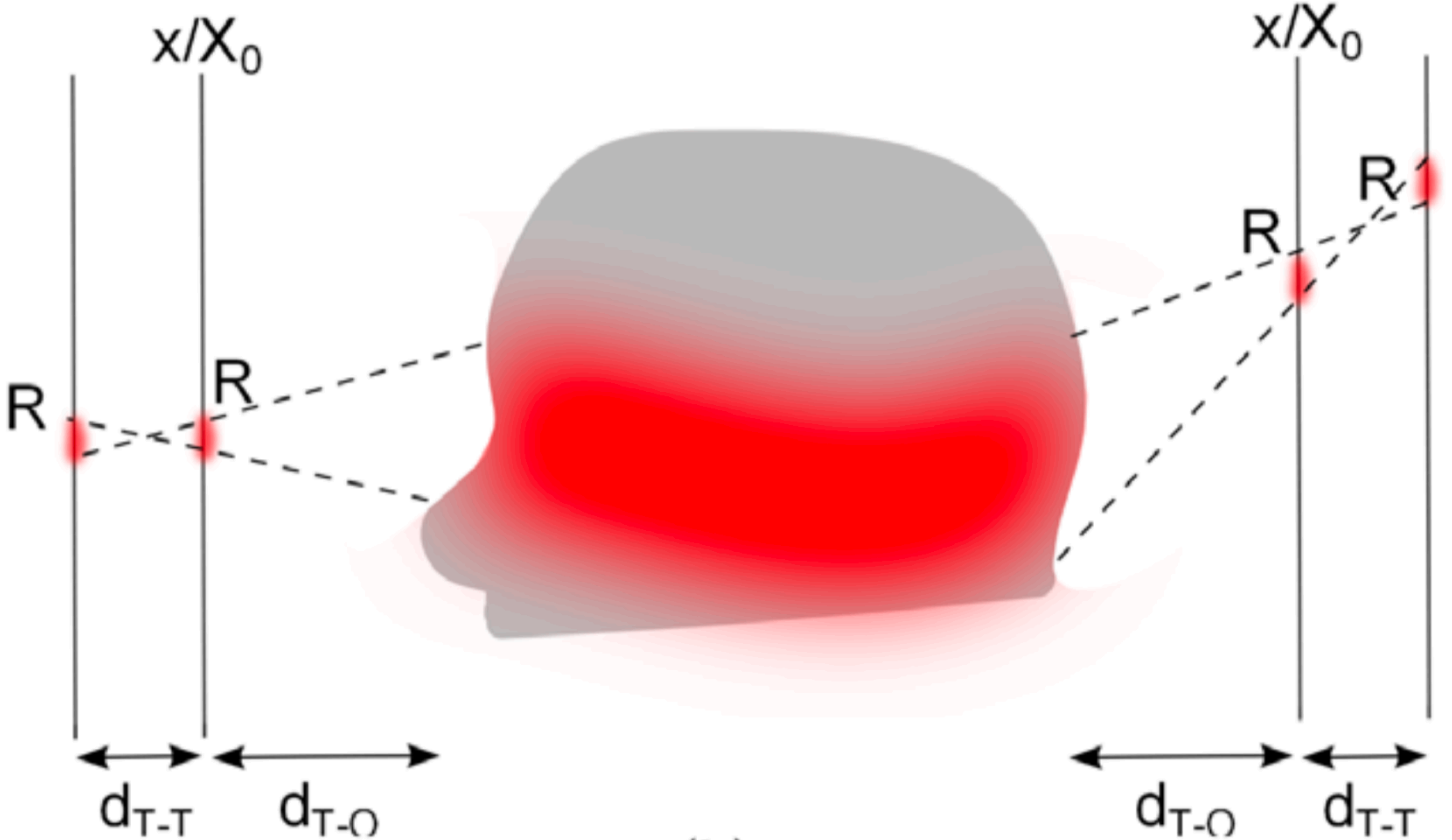


Figure from Bopp et al. 2014, PMB, DOI: 10.1088/0031-9155/59/23/N197

Likelihood of measurement process:

$$\Rightarrow L_{\text{meas}}(\tilde{y}_{\text{in}}, y_{\text{in}}) \propto \exp\left[-\frac{1}{2}(\tilde{y}_{\text{in}} - y_{\text{in}})^T \left(S_{\text{in}}\Sigma_{\text{in}}S_{\text{in}}^T\right)^{-1}(\tilde{y}_{\text{in}} - y_{\text{in}})\right]$$

$$\Rightarrow L_{\text{meas}}(\tilde{y}_{\text{out}}, y_{\text{out}}) \propto \exp\left[-\frac{1}{2}(\tilde{y}_{\text{out}} - y_{\text{out}})^T S_{\text{out}}^T \Sigma_{\text{out}}^{-1} S_{\text{out}}(\tilde{y}_{\text{out}} - y_{\text{out}})\right]$$

$$\Sigma_{\text{in}} = \sigma_p^2\, T_{\text{in}} \cdot T_{\text{in}}^T + \Sigma_{\text{sc}} \quad \text{and} \quad \Sigma_{\text{out}} = \sigma_p^2\, T_{\text{out}} \cdot T_{\text{out}}^T + \Sigma_{\text{sc}}$$

$$\Sigma_{\text{sc}} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_{\text{sc}}^2 \end{pmatrix} \quad \text{with} \quad \sigma_{\text{sc}} = \frac{13.6\text{ MeV}}{\beta(E)\,p(E)}\sqrt{\frac{x}{X_0}}\left[1 + 0.038\ln\left(\frac{x}{X_0}\right)\right]$$

$$T_{\text{out}} = \begin{pmatrix} 1 & 0 \\ -1/d_{\text{T}} & 1/d_{\text{T}} \end{pmatrix} \qquad T_{\text{in}} = \begin{pmatrix} 0 & 1 \\ -1/d_{\text{T}} & 1/d_{\text{T}} \end{pmatrix}$$

Ideal trackers

$$L(y_1, y_2 = y_{\text{out}} | y_{\text{in}}) = L_{\text{scat}}(y_{\text{in}} \to y_1) \times L_{\text{scat}}(y_1 \to y_2 = y_{\text{out}} | y_{\text{in}})$$

Trackers with uncertainties

$$L(y_1, y_2 = \tilde{y}_{\text{out}} | \tilde{y}_{\text{in}}) = \int L_{\text{meas}}(\tilde{y}_{\text{in}}, y_{\text{in}}) L_{\text{scat}}(y_{\text{in}} \to y_1) \mathrm{d}y_{\text{in}} \times \int L_{\text{scat}}(y_1 \to y_2 = y_{\text{out}}) L_{\text{meas}}(\tilde{y}_{\text{out}}, y_{\text{out}}) \mathrm{d}y_{\text{out}}$$

MLP considering tracker uncertainties

$$y_{\mathrm{MLP}}(u) = C_2\left(C_1 + C_2\right)^{-1} R_0 S_{\mathrm{in}} \cdot \tilde{y}_{\mathrm{in,d}} + C_1\left(C_1 + C_2\right)^{-1} R_1^{-1} S_{\mathrm{out}}^{-1} \cdot \tilde{y}_{\mathrm{out,d}}$$

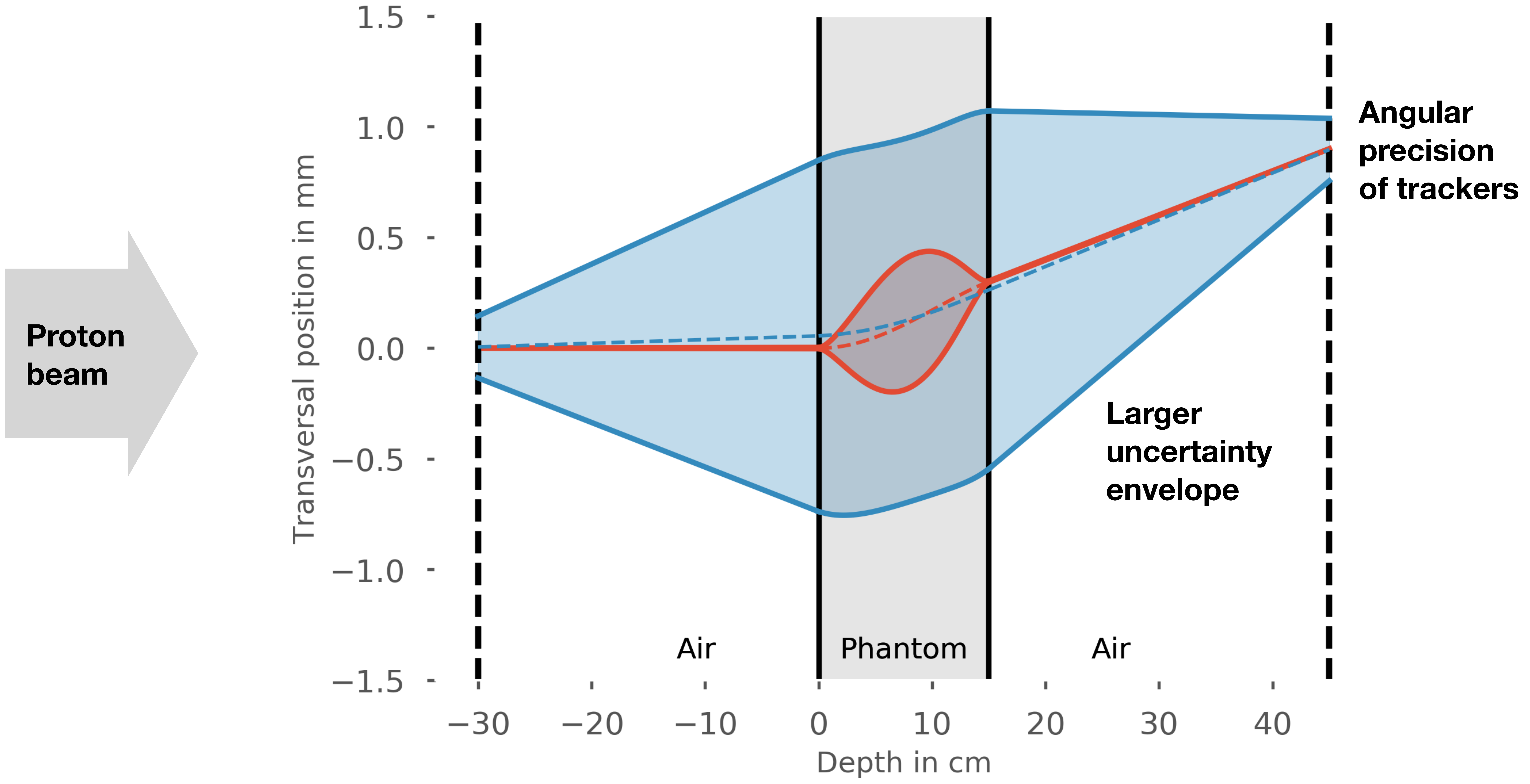
$$\Sigma_{\mathrm{MLP}}(u) = C_1\left(C_1 + C_2\right)^{-1} C_2$$

$$\sigma_{\mathrm{MLP}}(u) = \left(\Sigma_{\mathrm{MLP}}(u)\right)_{1,1}$$

with $$C_1 = R_0 S_{\mathrm{in}} \Sigma_{\mathrm{in}} S_{\mathrm{in}}^T R_0^T + \Sigma_1$$

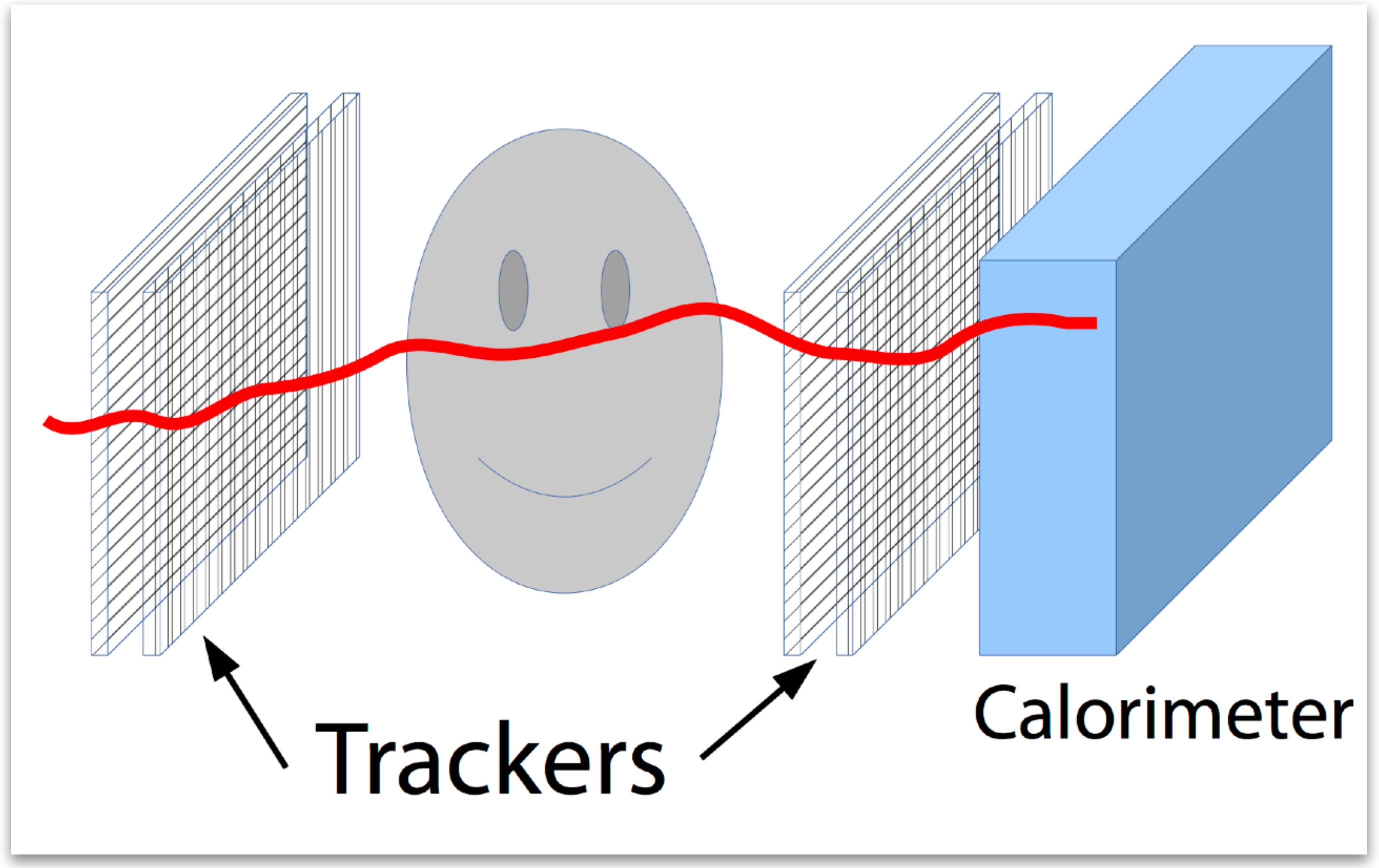
$$C_2 = R_1^{-1} S_{\mathrm{out}}^{-1} \Sigma_{\mathrm{out}} (S_{\mathrm{out}}^{-1})^T (R_1^{-1})^T + R_1^{-1} \Sigma_2 (R_1^{-1})^T$$

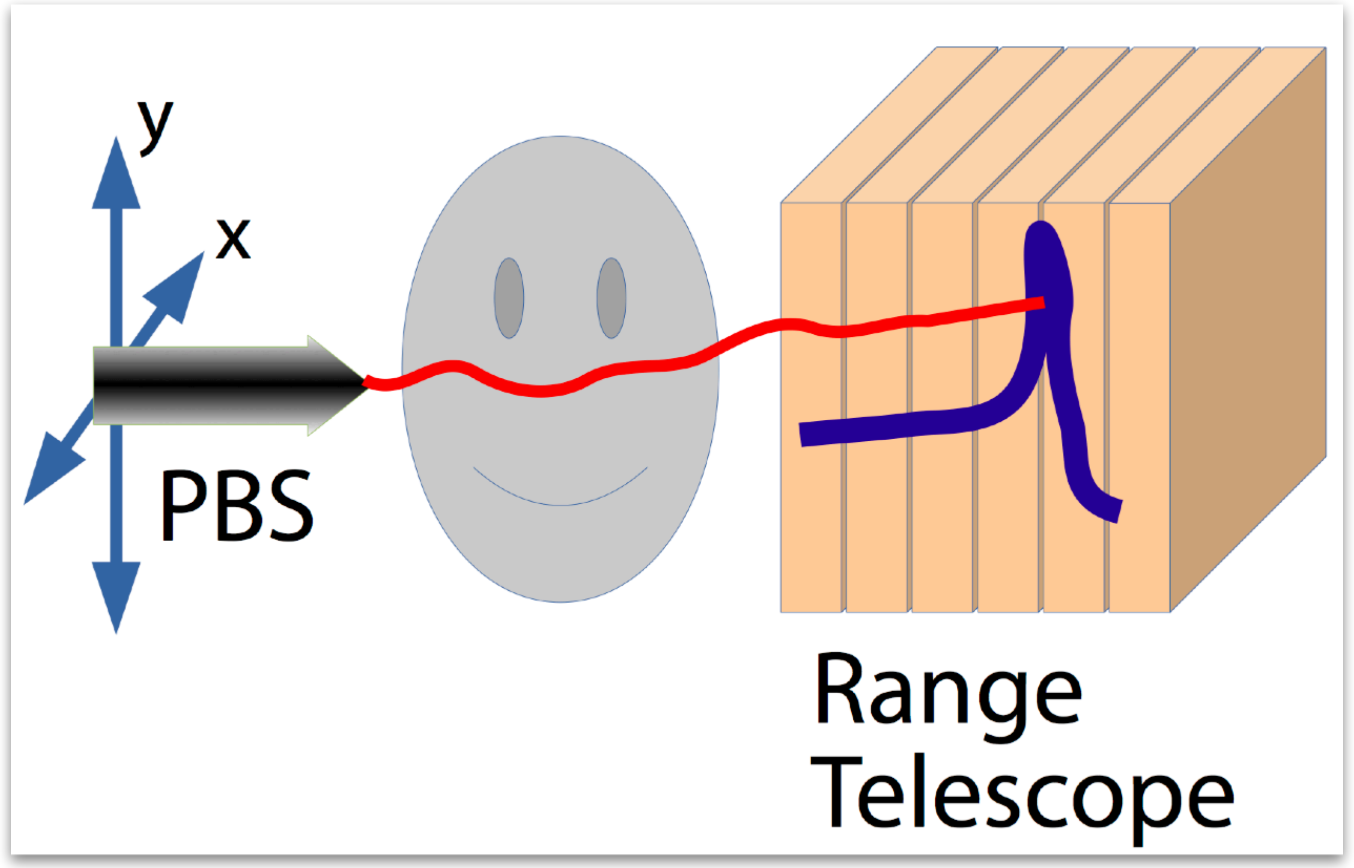
Tracker uncertainties

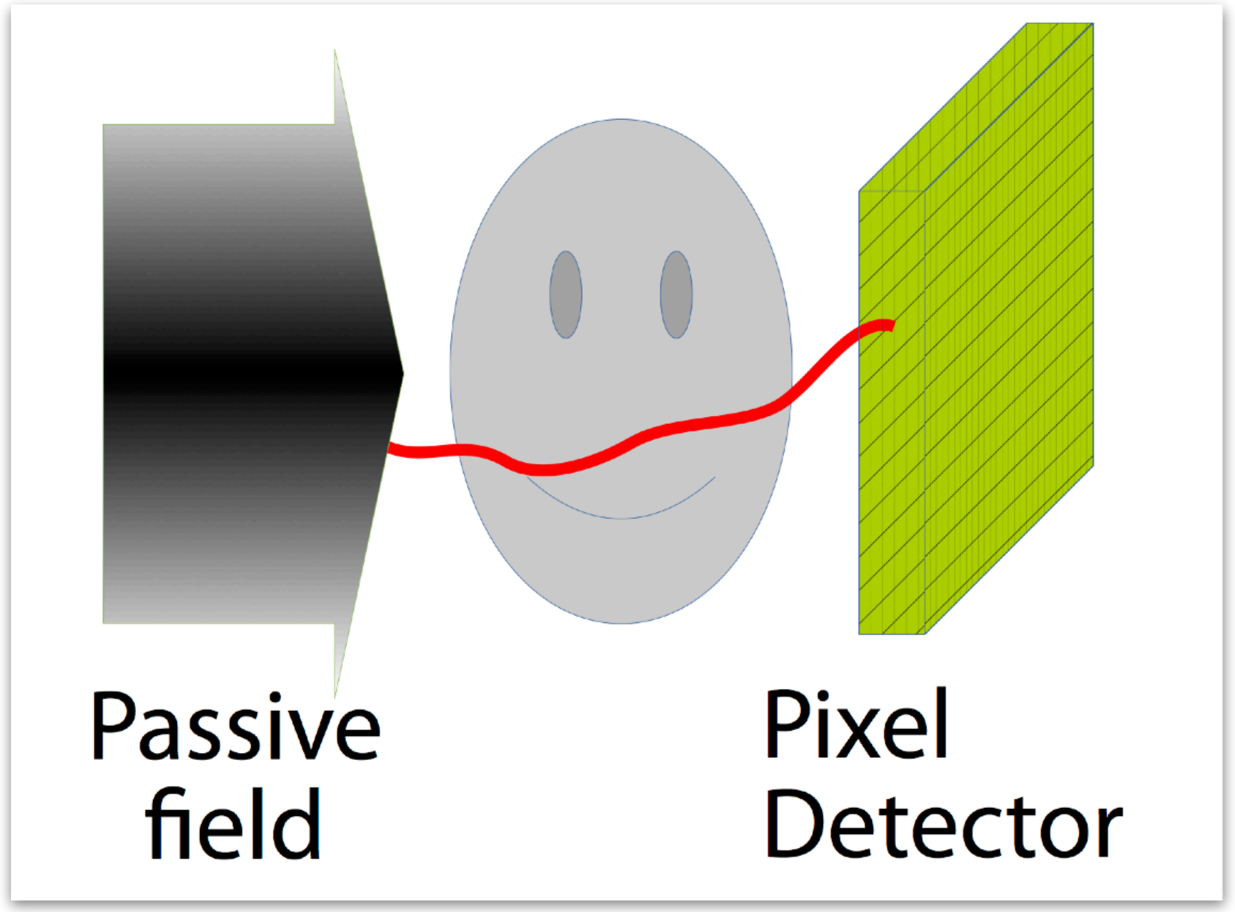


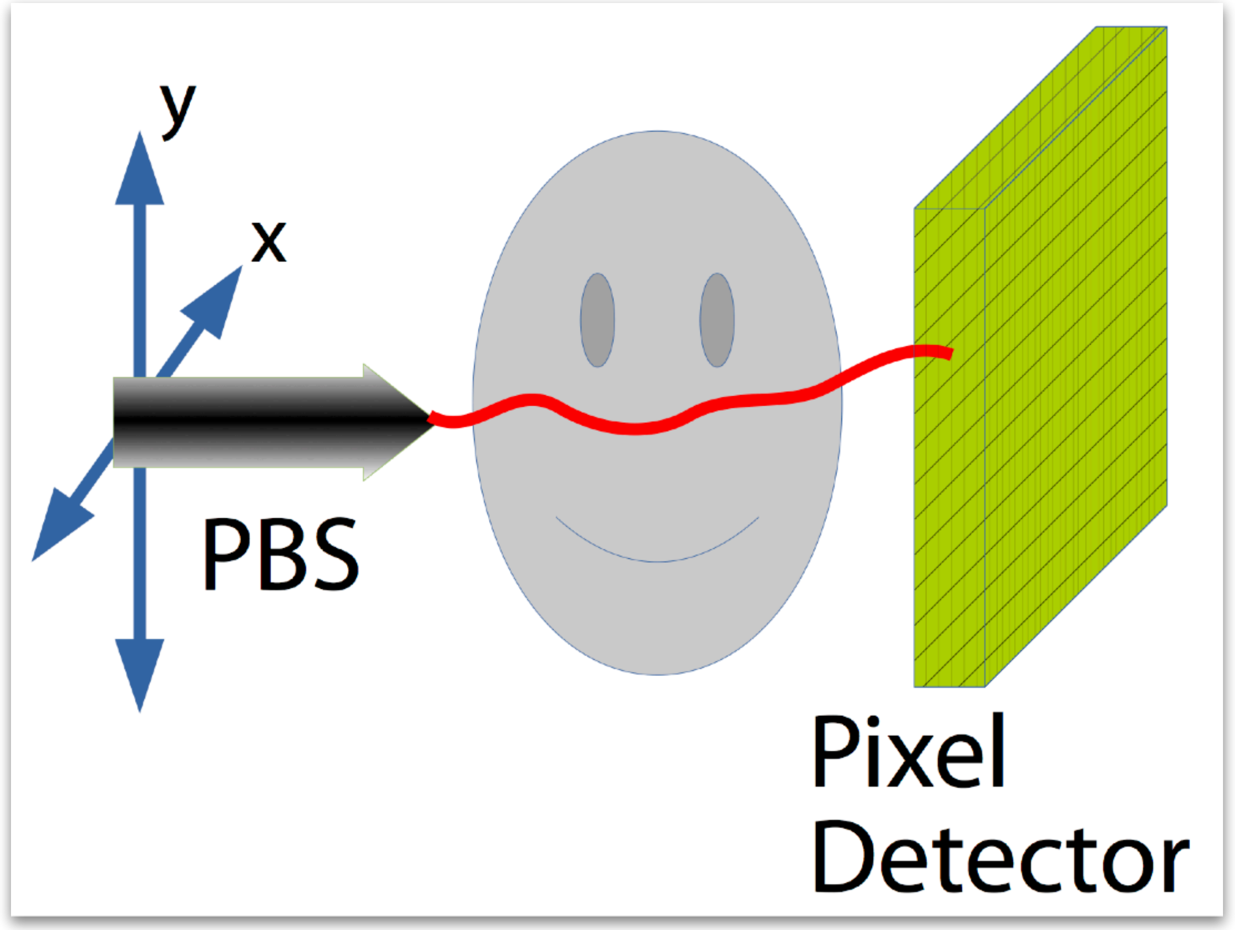
Bopp et al. 2014, PMB, DOI: 10.1088/0031-9155/59/23/N197
Penfold S 2011, Radiat. Meas. 46

Apply formalism to integral mode set-ups

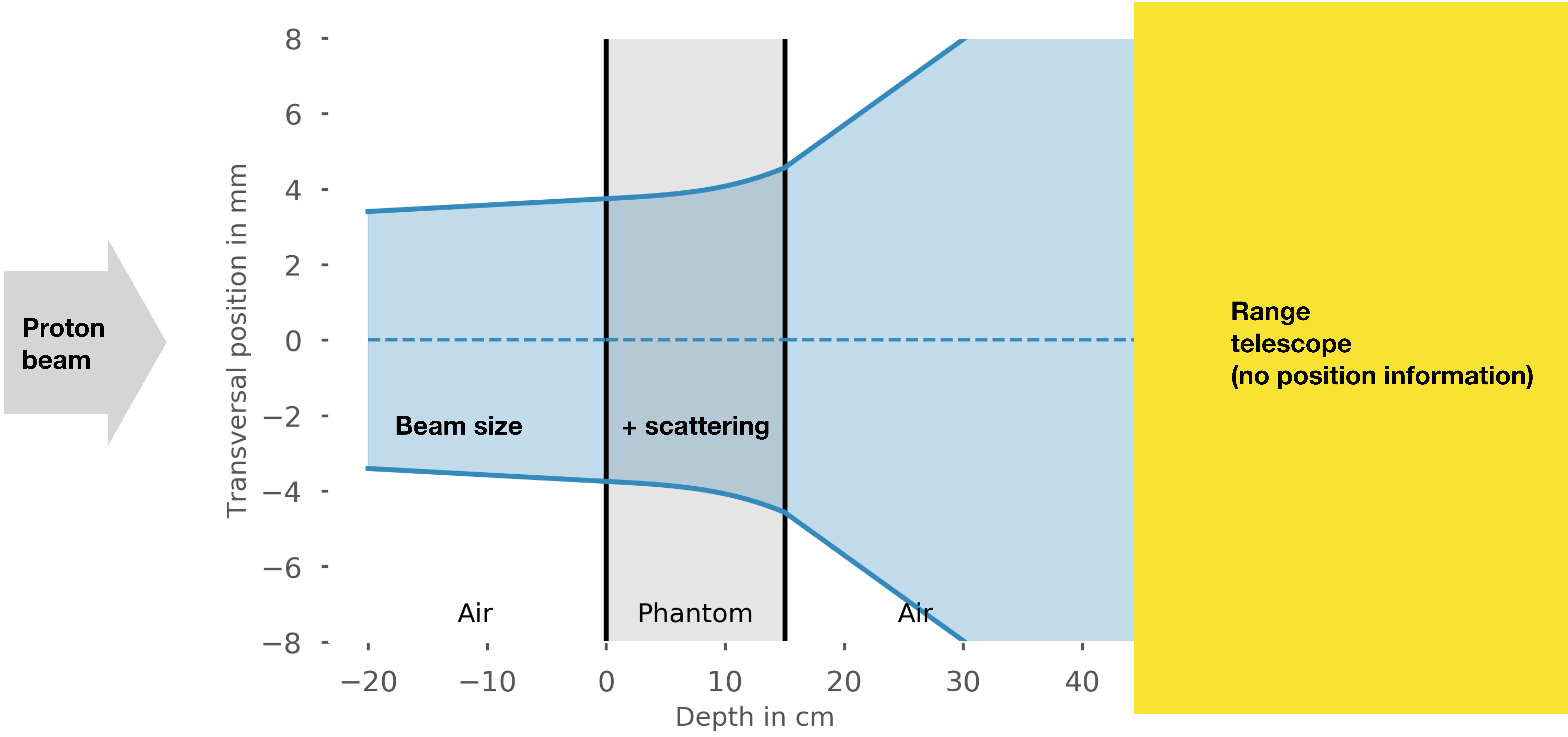




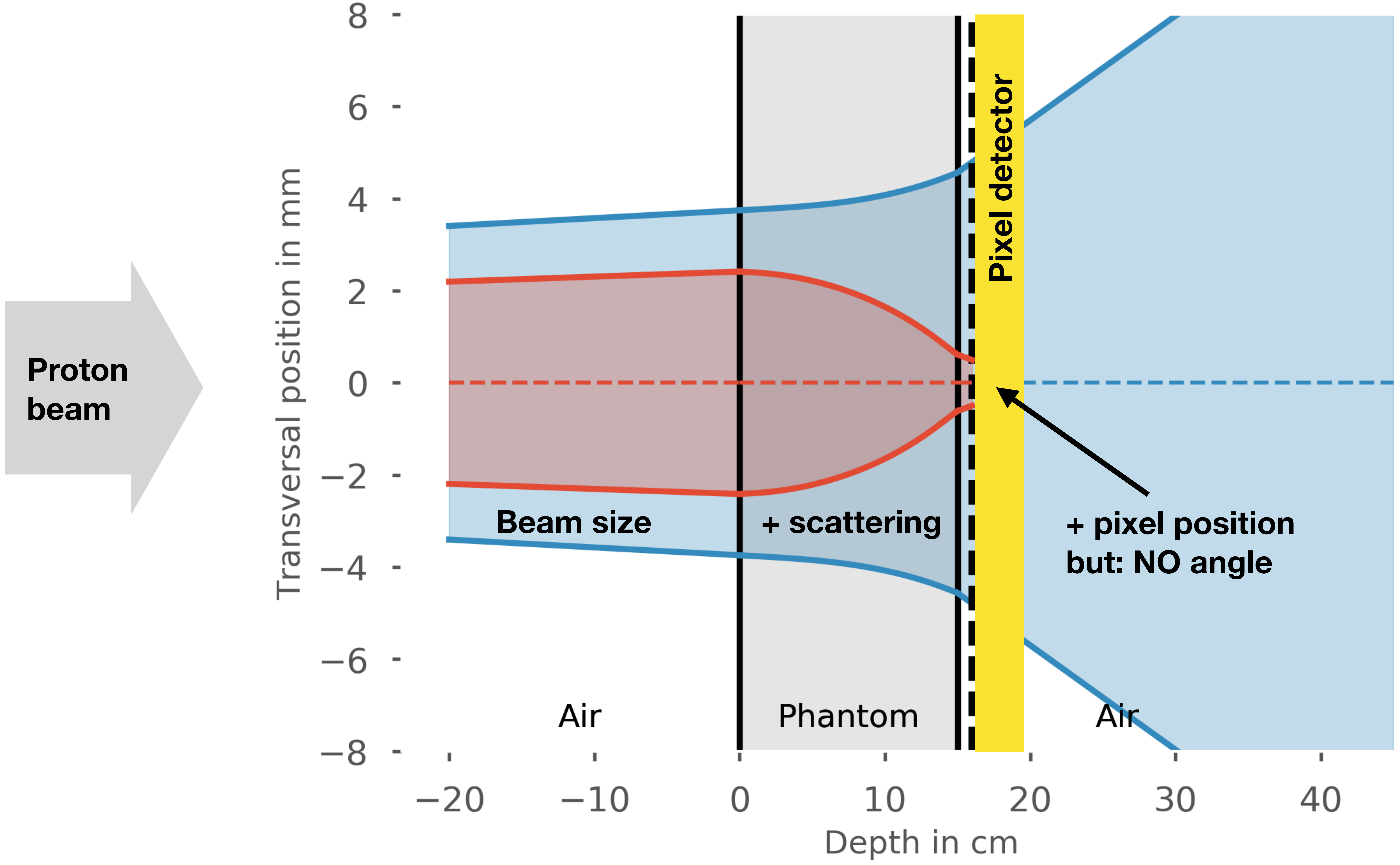




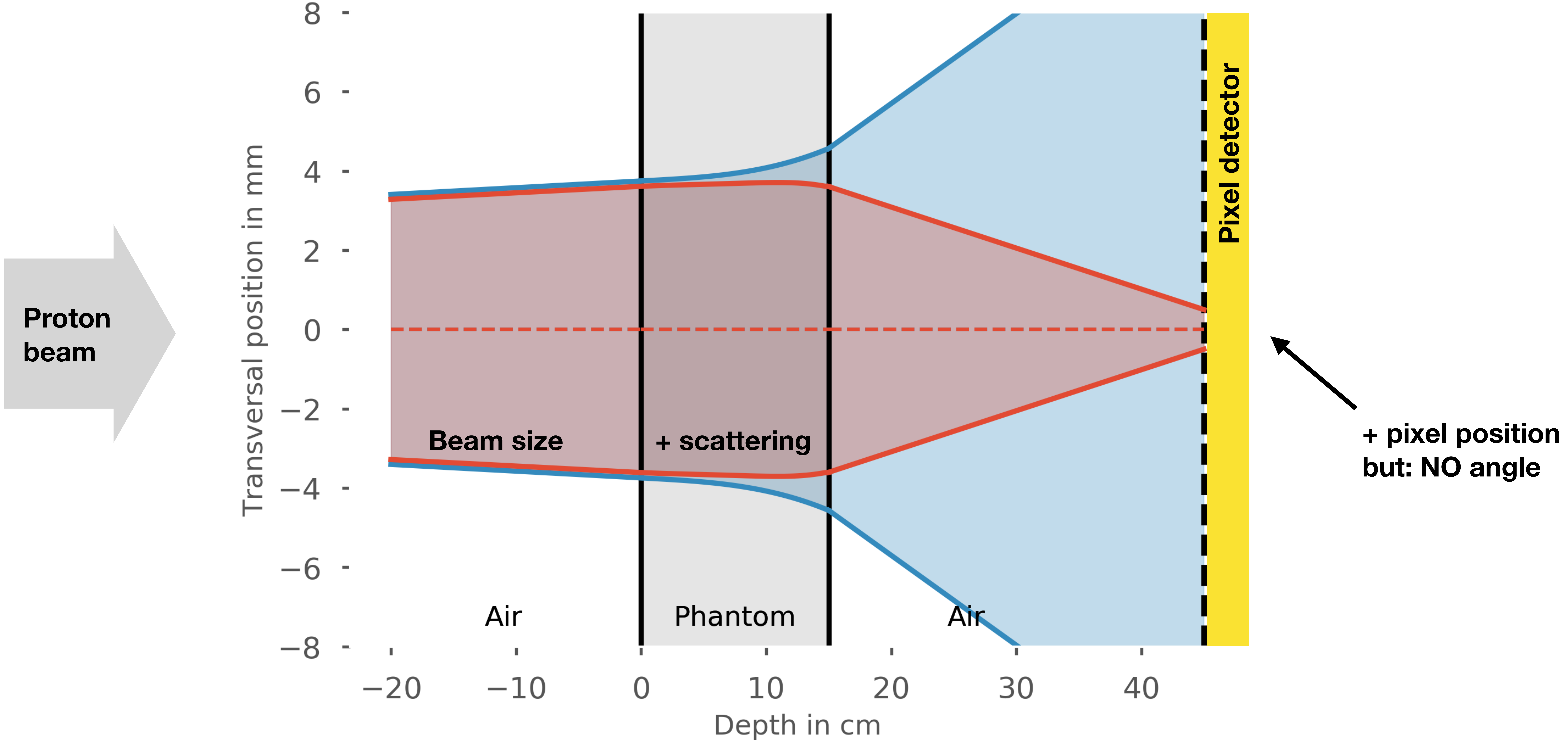
PBS-based set-ups



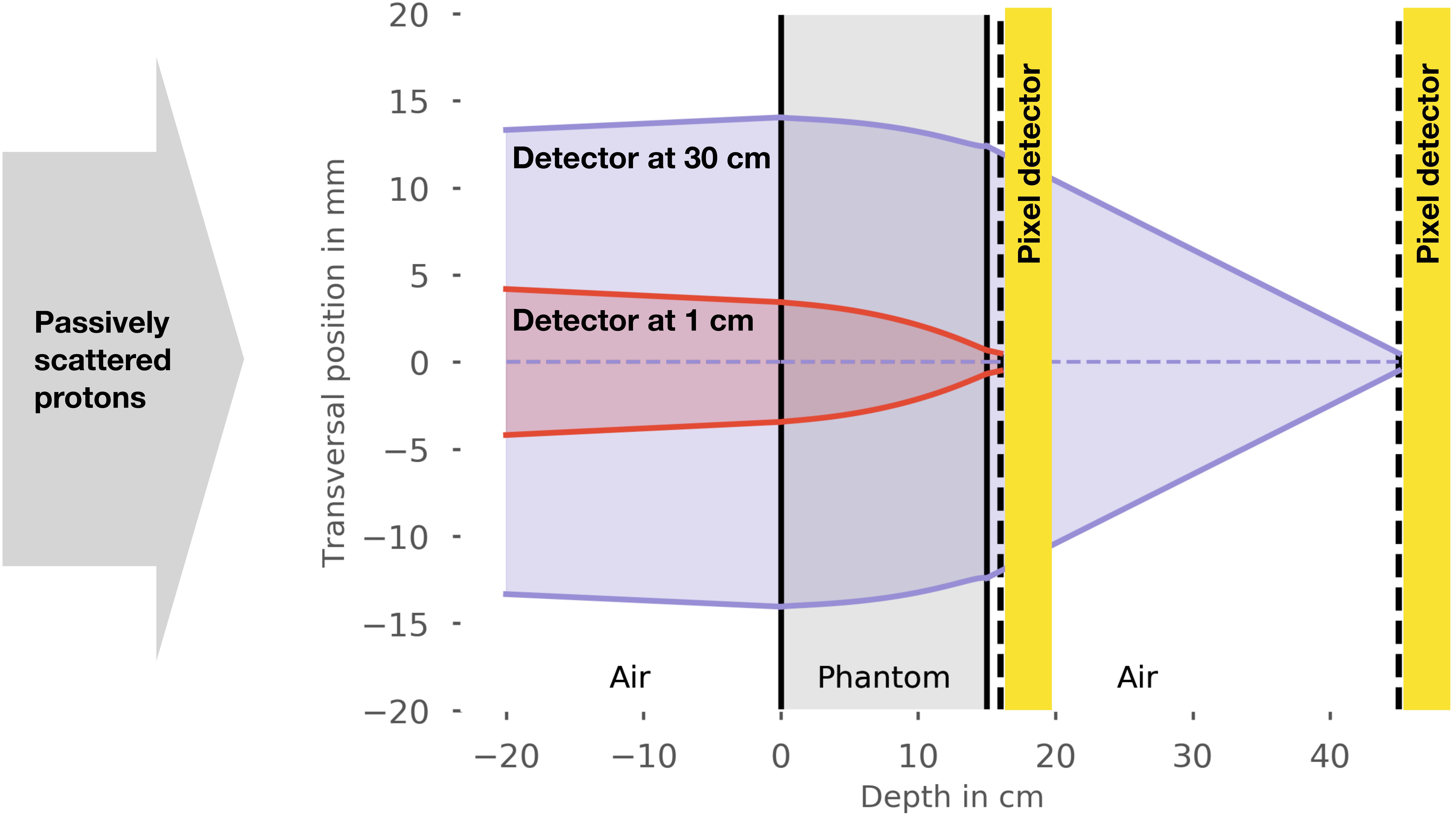
PBS-based set-ups



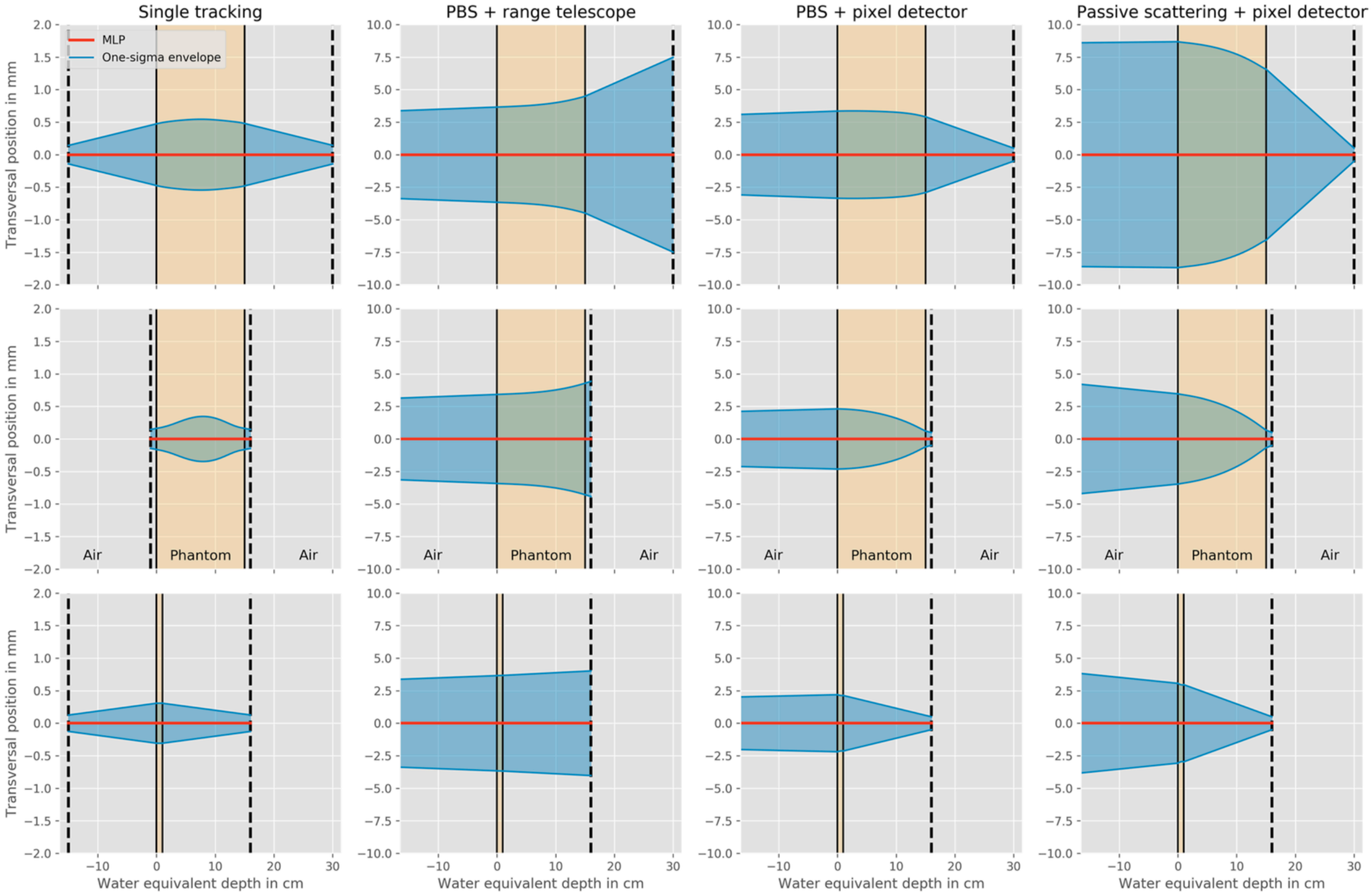
PBS-based set-ups



Passive scattering set-ups

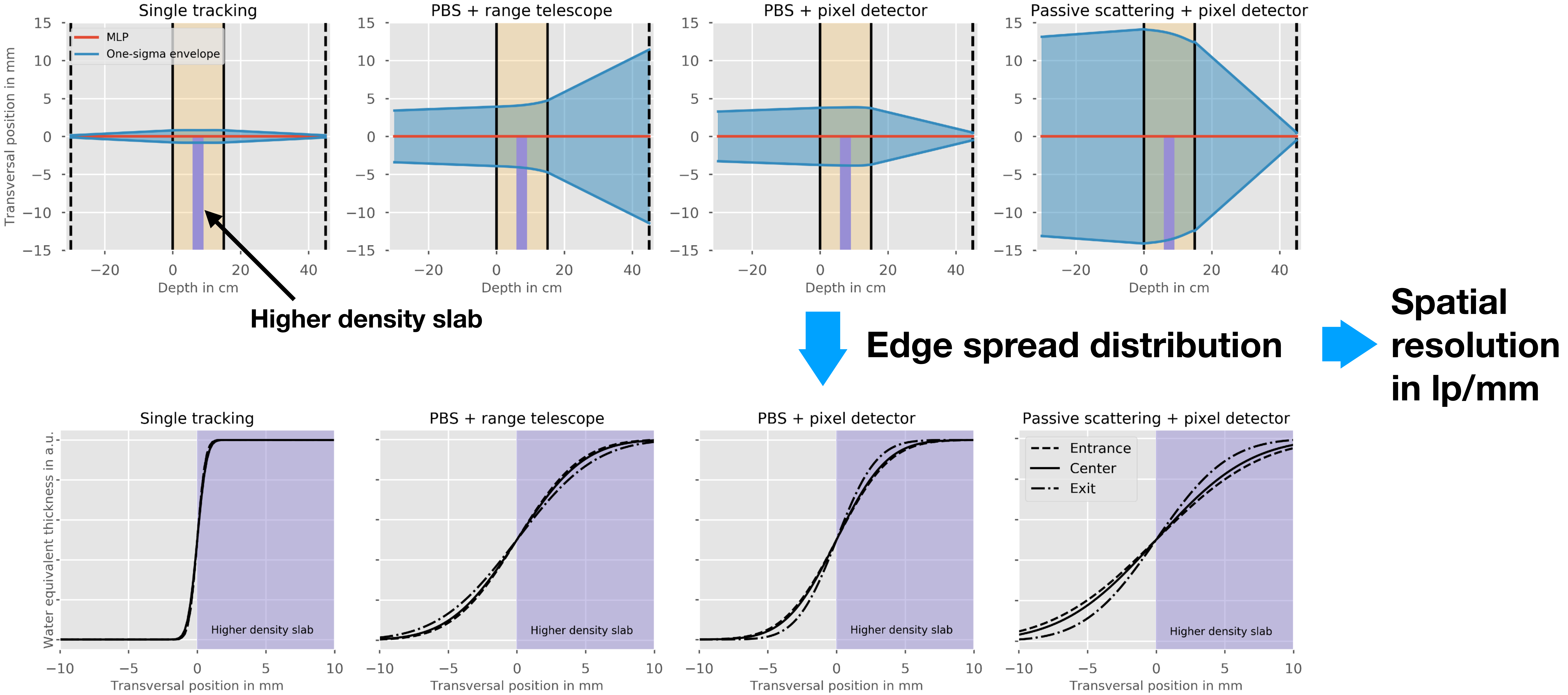


Set-up parameter	$\sigma_{t_{in}}$	$\sigma_{\theta_{in}}$	$\sigma_{t_{out}}$	$\sigma_{\theta_{out}}$
Single tracking idealised	0	0	0	0
PBS + range telescope idealised	0	0	$\rightarrow \infty$	$\rightarrow \infty$
PBS + pixel detector idealised	0	0	0	$\rightarrow \infty$
Passive scattering + pixel detector idealised	$\rightarrow \infty$	0	0	$\rightarrow \infty$
Single tracking (see section 28)	0.15 mm	3 mrad	0.15 mm	3 mrad
Single tracking without angle measurement	0.5 mm	15 mrad	0.5 mm	$\rightarrow \infty$ (45°)
PBS + range telescope	8/2.35 mm	0.1 mrad	20 cm	$\rightarrow \infty$ (45°)
passive scattering + pixel detector	20 cm	15 mrad	0.5 mm	$\rightarrow \infty$ (45°)
PBS + pixel detector	8/2.35 mm	0.1 mrad	0.5 mm	$\rightarrow \infty$ (45°)

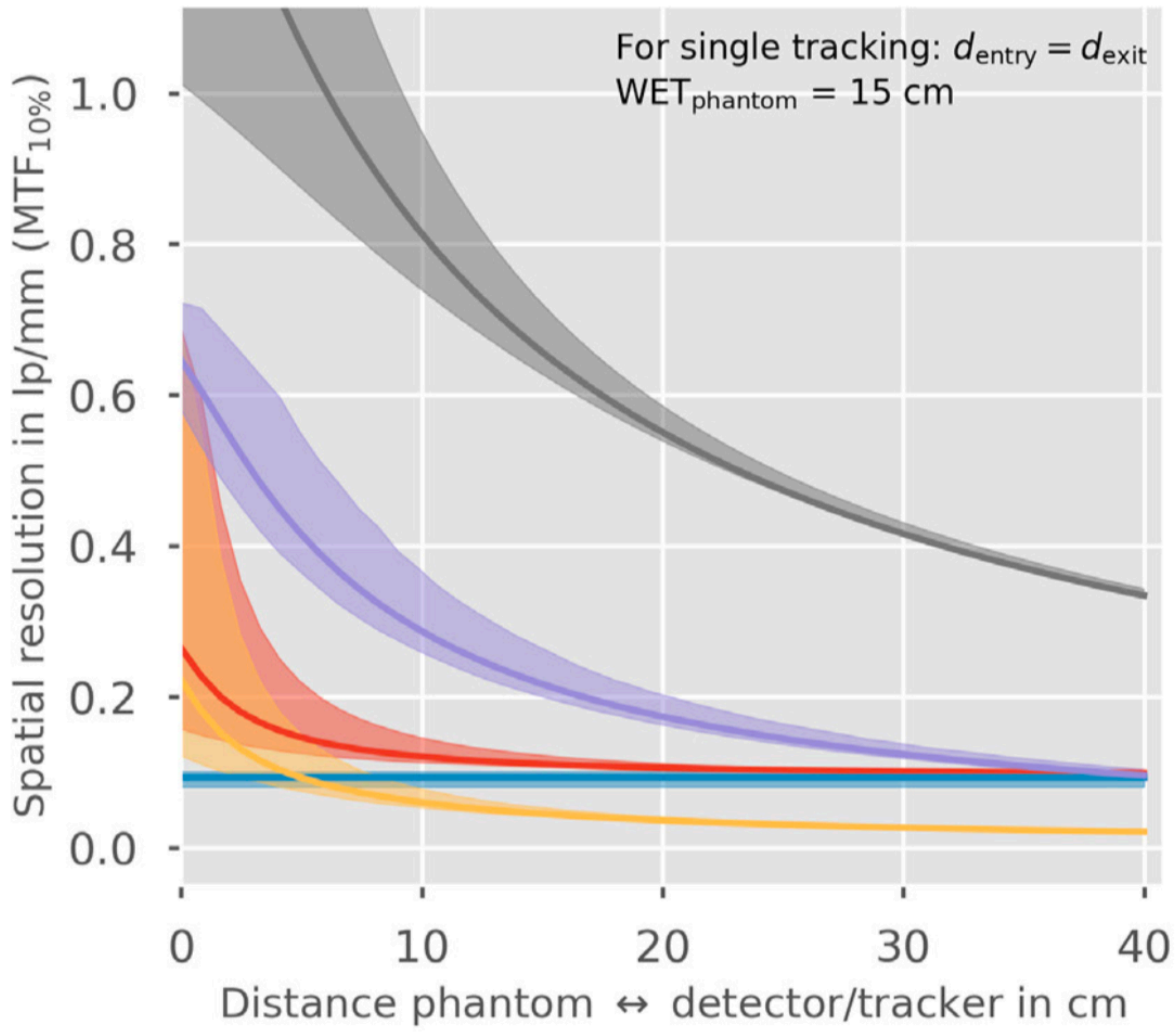


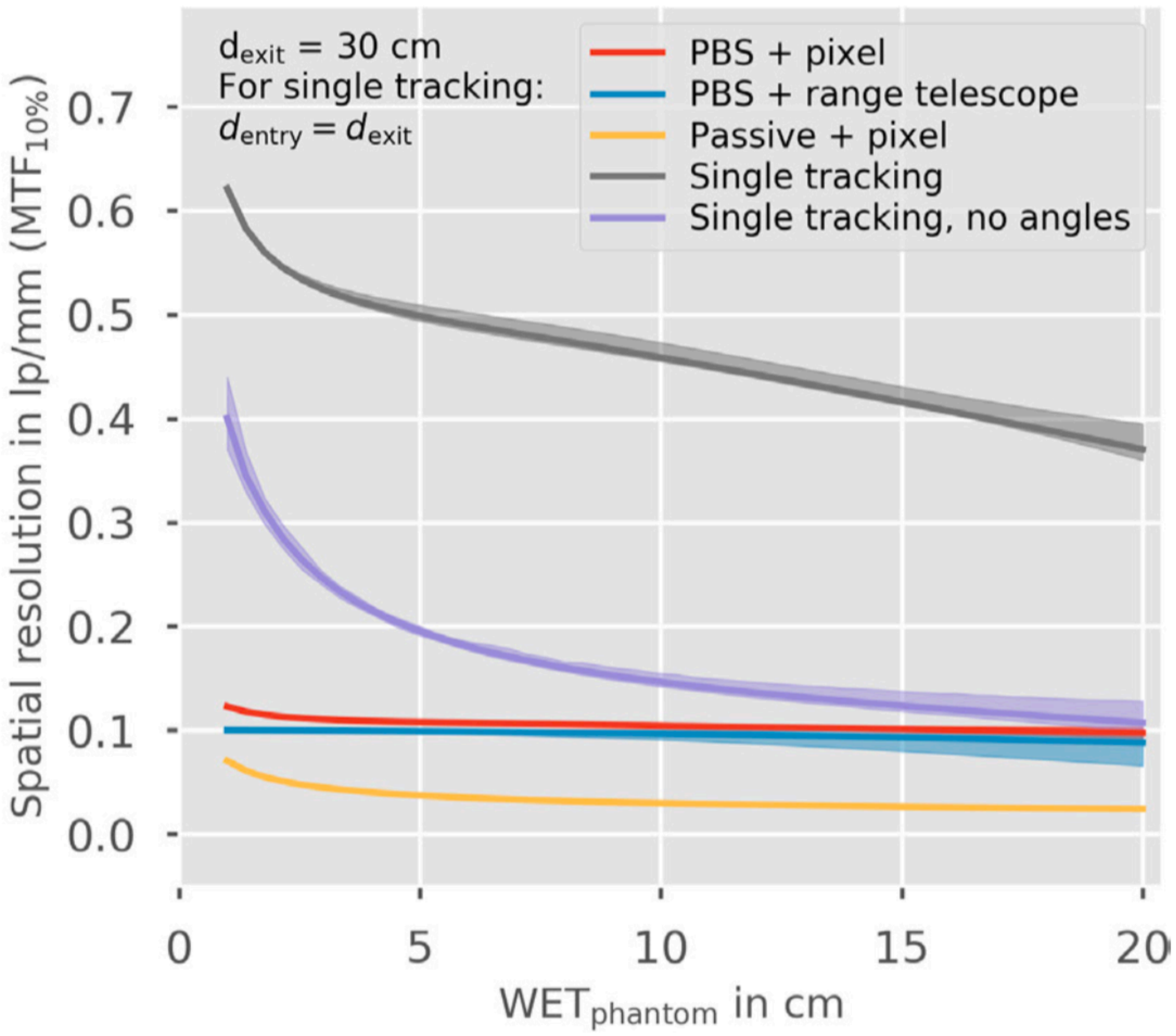
Single tracking
PBS + range telescope
PBS + pixel detector
Passive scattering + pixel detector
MLP
One-sigma envelope
Transversal position in mm
Air
Phantom
Air
Water equivalent depth in cm

Measure of spatial resolution

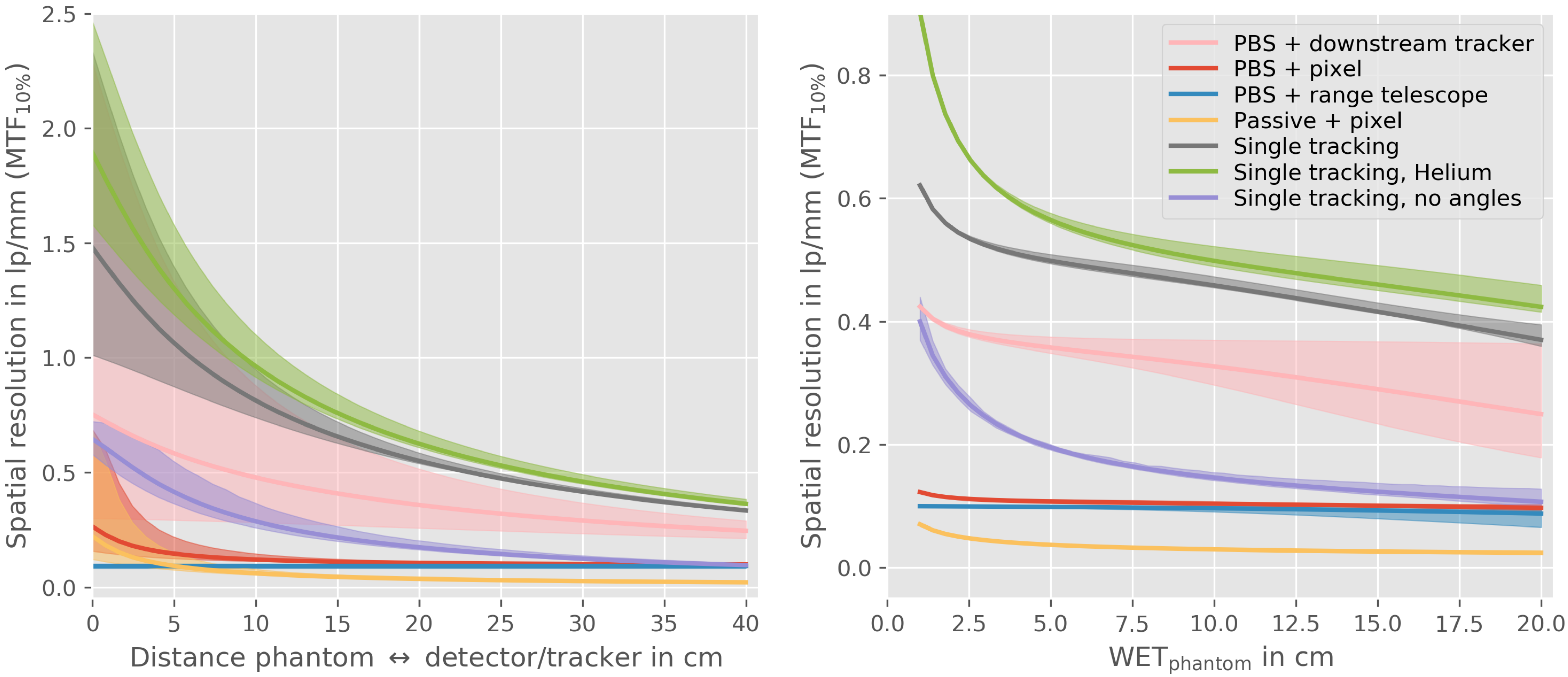


Impact of detector distance on spatial resolution



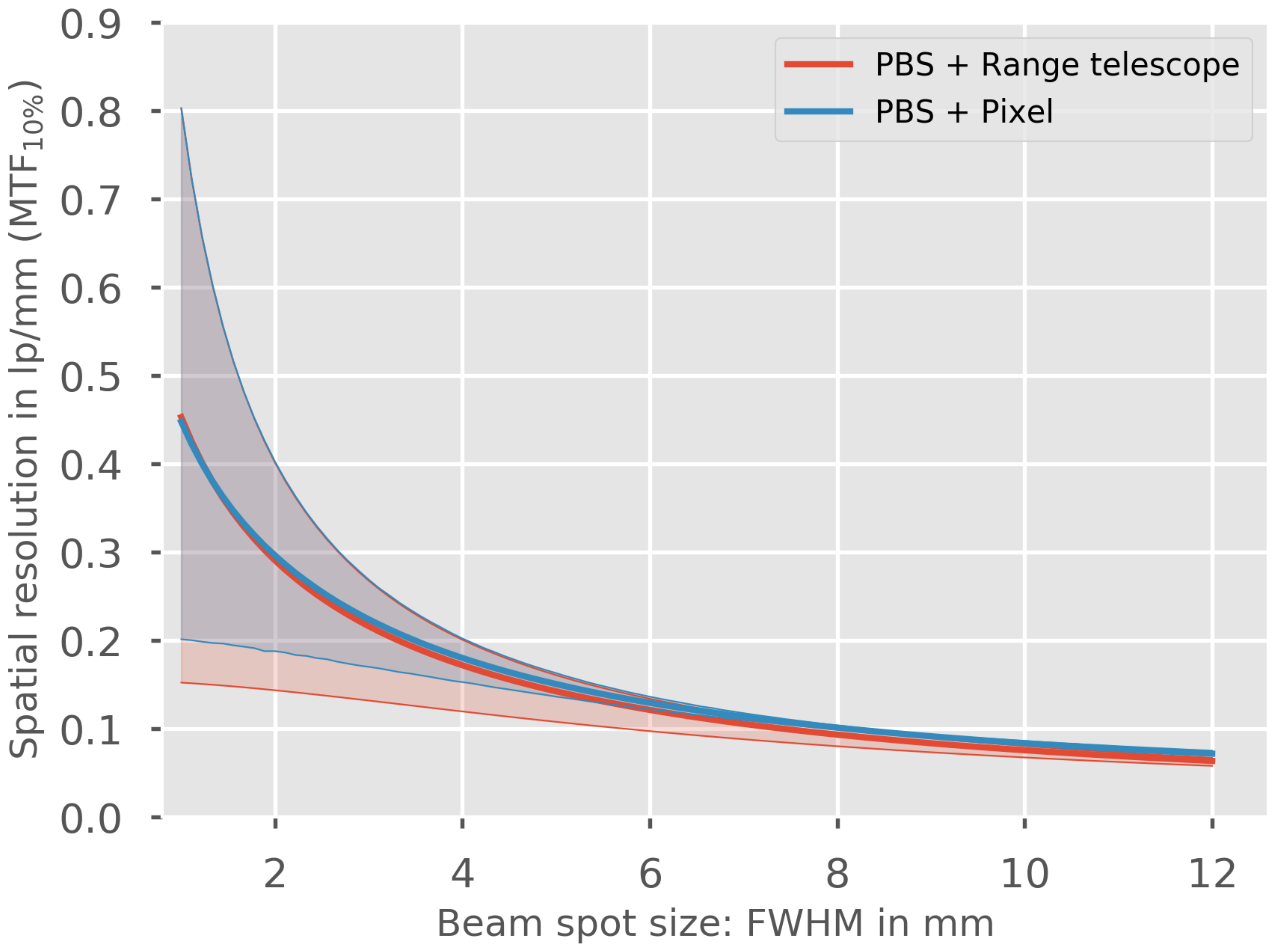


How about Helium and the Bergen set-up?



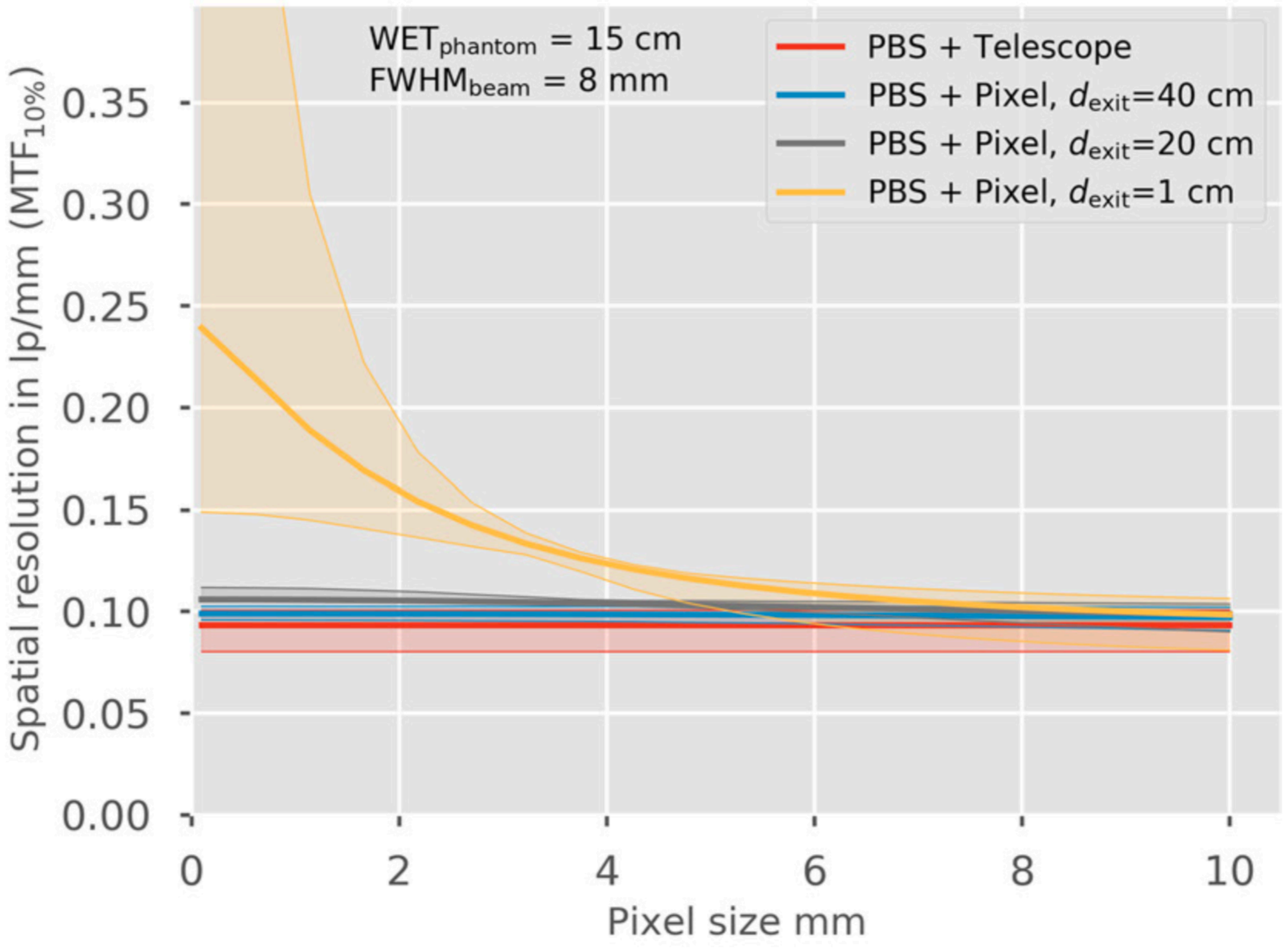
***preliminary**

Impact of beam size in PBS set-ups



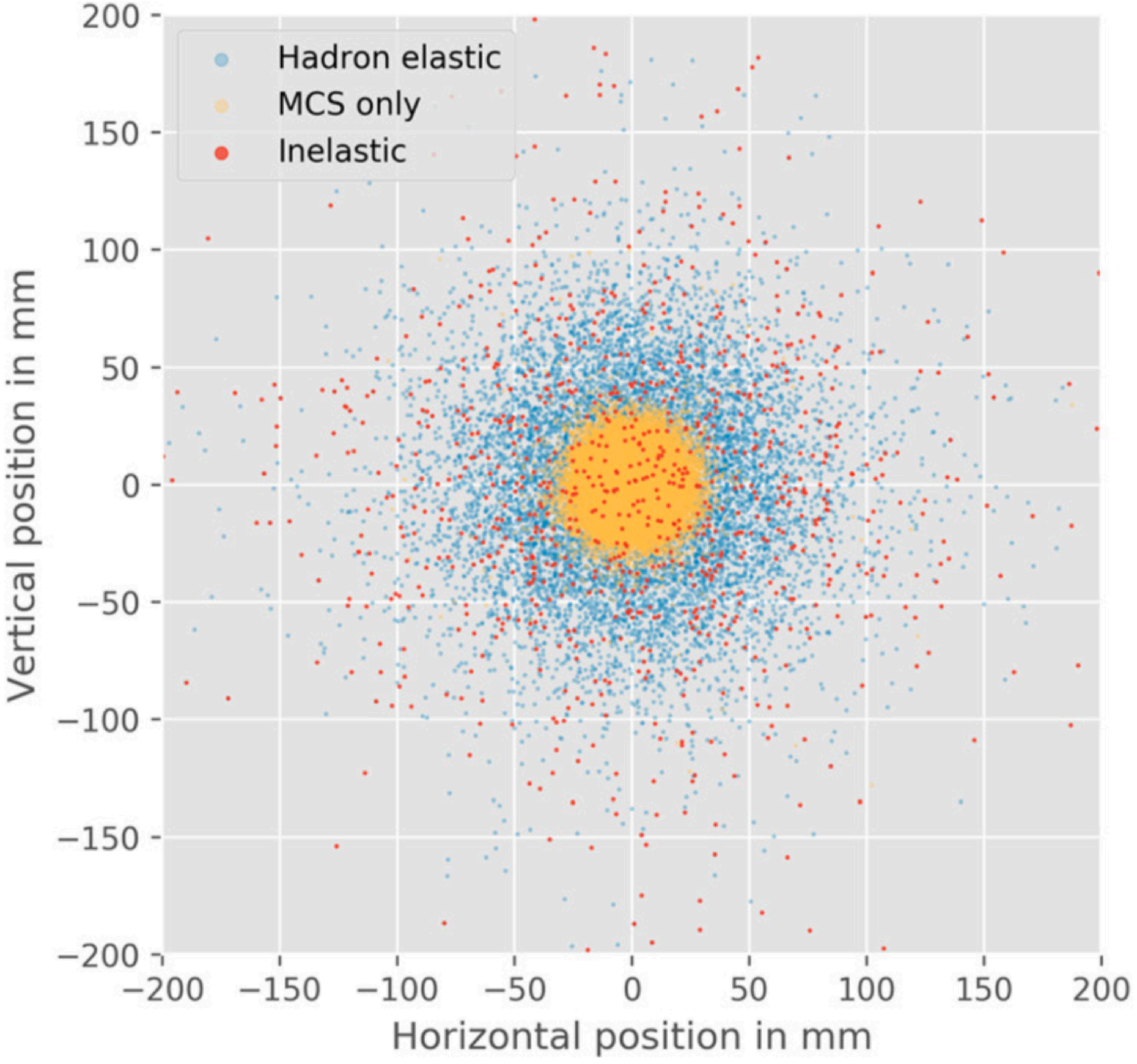
Small beam size greatly improves spatial resolution

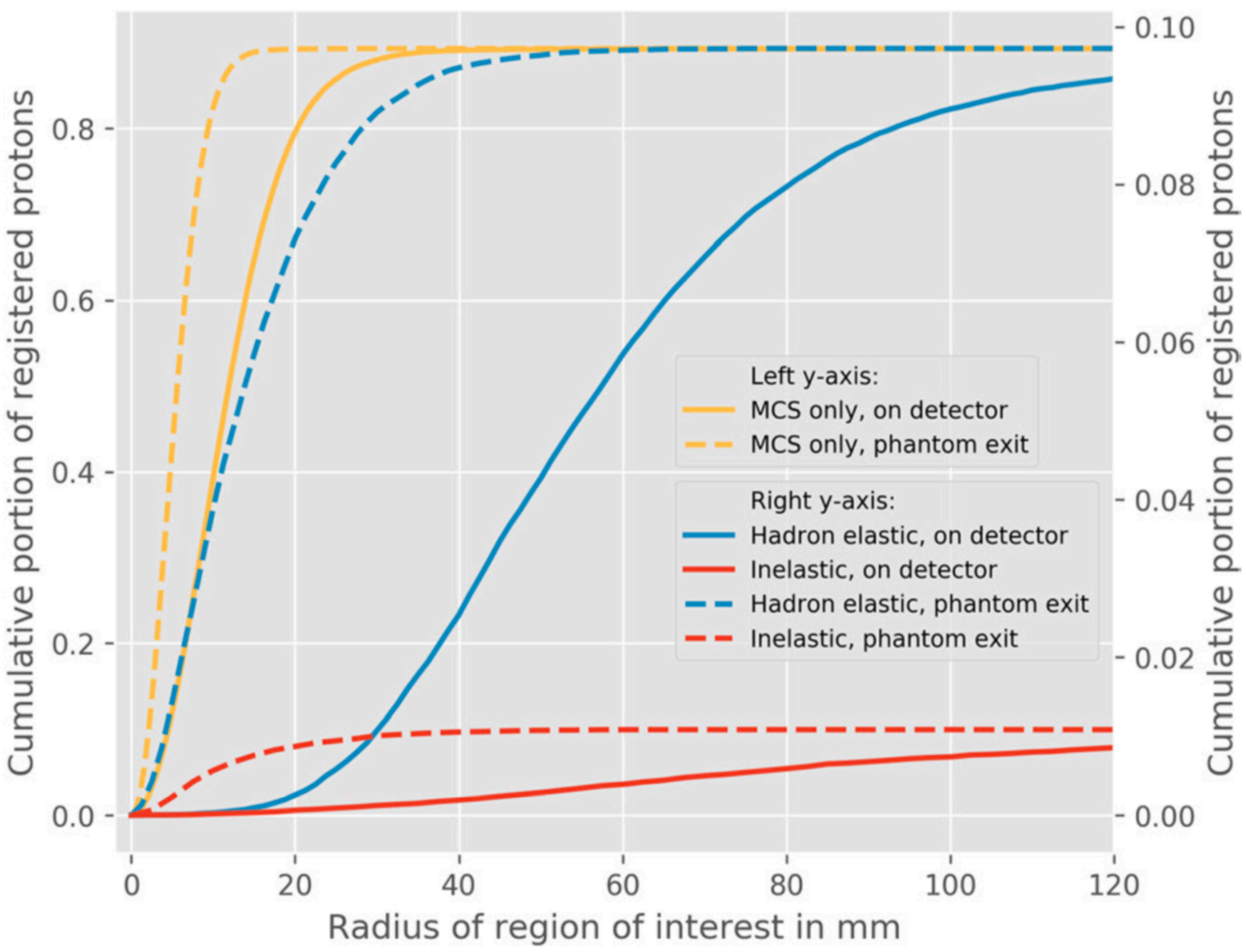
Pixel size



... does not really matter unless the detector is placed very close yo the phantom

So pixels are useless?



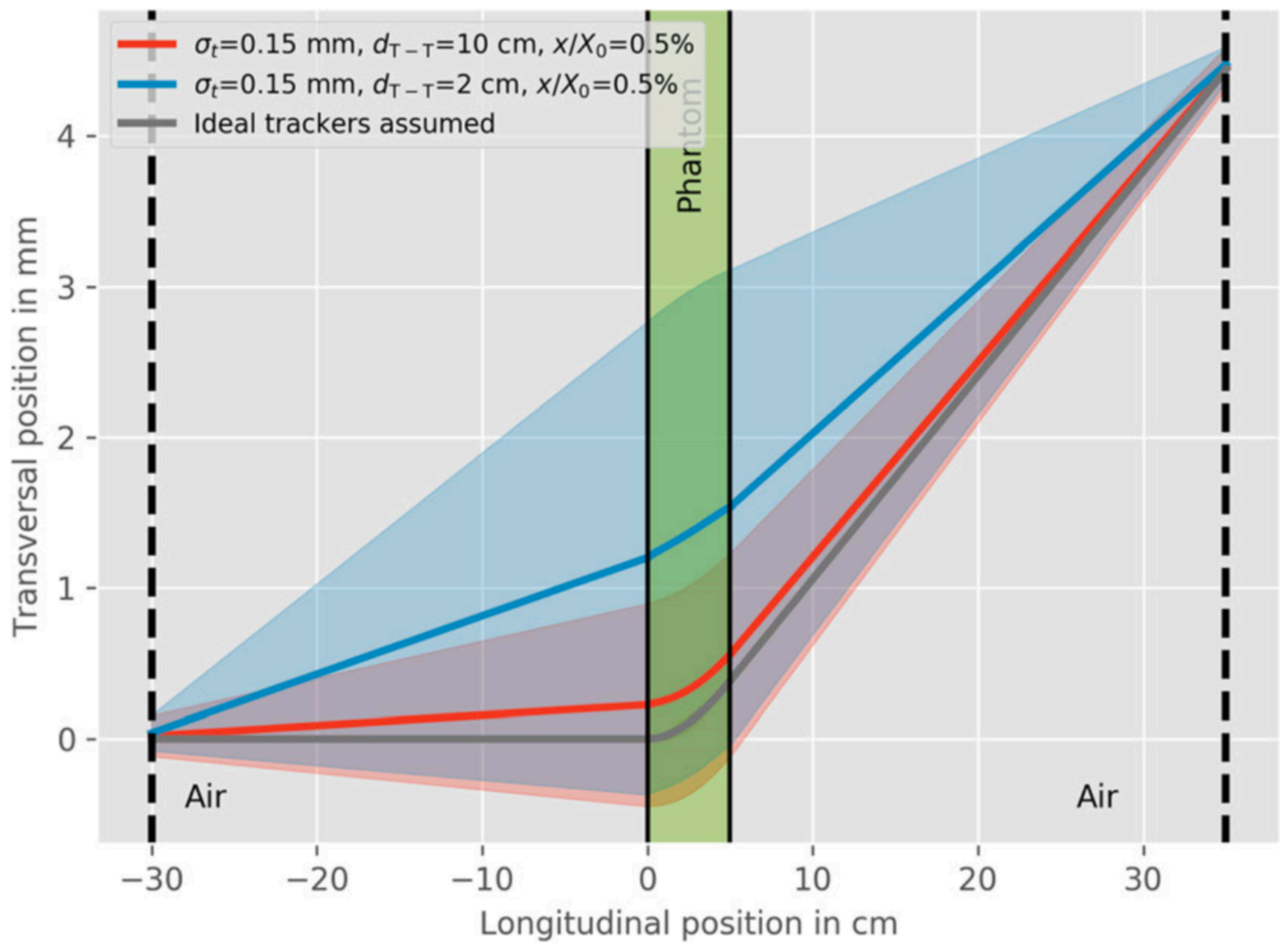


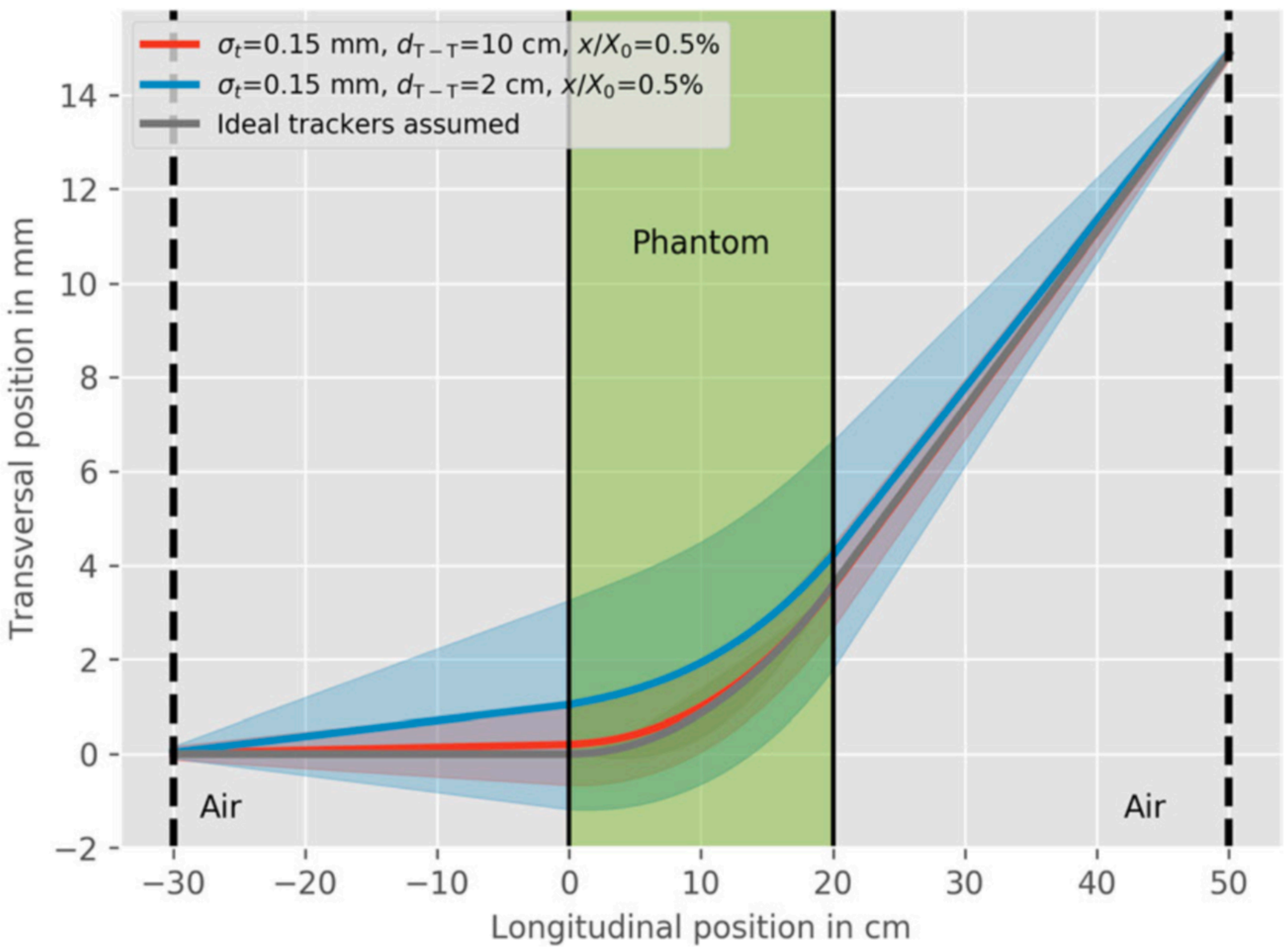
Impact of experimental uncertainties on MLP

$$y_{\mathrm{MLP}}(u) = C_2\left(C_1 + C_2\right)^{-1} R_0 S_{\mathrm{in}} \cdot \tilde{y}_{\mathrm{in,d}} + C_1\left(C_1 + C_2\right)^{-1} R_1^{-1} S_{\mathrm{out}}^{-1} \cdot \tilde{y}_{\mathrm{out,d}}$$

$$C_1 = R_0 S_{\mathrm{in}} \Sigma_{\mathrm{in}} S_{\mathrm{in}}^T R_0^T + \Sigma_1$$

$$C_2 = R_1^{-1} S_{\mathrm{out}}^{-1} \Sigma_{\mathrm{out}} (S_{\mathrm{out}}^{-1})^T (R_1^{-1})^T + R_1^{-1} \Sigma_2 (R_1^{-1})^T$$





Conclusion

- List mode:
  tracker uncertainty very important
  spatial resolution degrades drastically with tracker distance
- Passive field: Must put detector close to phantom/patient
- PBS: Small beam size improves spatial resolution a lot
- Pixel size is irrelevant for spatial resolution
  but pixels allows for region of interest filtering
- Refine MLP estimation?

Merci !