A short refresher on filtered backprojection reconstruction for pCT

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CREATIS

2D Radon transform

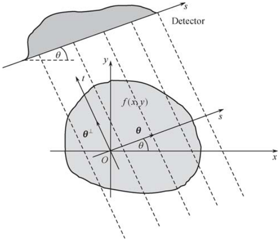


Figure reprinted from G.L. Zeng. *Medical Image Reconstruction*. Springer, 2010

$$
\begin{aligned}
& p(s,\theta) \\
& = p_\theta(s) \\
& = \int_{\mathbb{R}} f(s\cos\theta - t\sin\theta, s\sin\theta + t\cos\theta)\,\mathrm{d}t
\end{aligned}
$$

Fourier / Central slice theorem (in 2D)

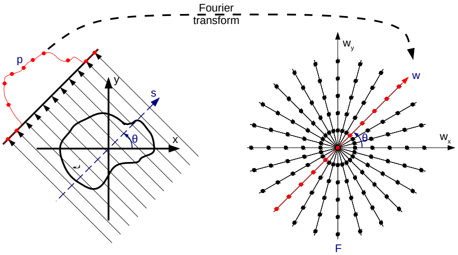
Noting the 1D and 2D Fourier transforms of p_θ and f

$$P_\theta(w) = \int_{-\infty}^{+\infty} p_\theta(s)\, \mathrm{e}^{-2\pi i s w}\, \mathrm{d}s$$

$$F(w_x, w_y) = \iint_{\mathbb{R}^2} f(x, y)\, \mathrm{e}^{-2\pi i (x w_x + y w_y)}\, \mathrm{d}x\, \mathrm{d}y$$

Theorem

$$P_\theta(w) = F(w\cos\theta, w\sin\theta)$$



2D inverse Radon transform[1]

$$f(x,y) = \int_0^{\pi} \int_{\mathbb{R}} P_\theta(w)|w|e^{2\pi i w(x\cos\theta + y\sin\theta)}\,\mathrm{d}w\,\mathrm{d}\theta$$

1. Ramp filter the projections,
2. Backprojection of the filtered projections.

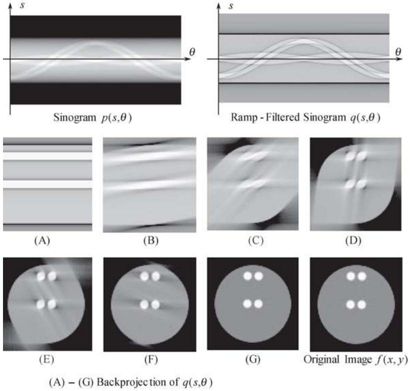


Figure reprinted from G.L. Zeng. *Medical Image Reconstruction*. Springer, 2010

[1] J. Radon. "On the determination of functions from their integral values along certain manifolds". In: *IEEE Transactions on Medical Imaging* 5.4 (1986), pp. 170–176.

From 2D inverse Radon transform to...

... fan-beam CT[2]: change of variable.

... FDK algorithm[3]: approximate algorithm (except in the central slice) based on 2D fan-beam CT.

In both algorithms:

- same ramp filter,
- additional weighting (before filtering and during backprojection).

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[2] A.C. Kak and M. Slaney. *Principles of computerized tomographic imaging*. IEEE Press, 1988.

[3] L.A. Feldkamp, L.C. Davis, and J.W. Kress. "Practical cone-beam algorithm". In: *J Opt Soc Am A* 1.6 (1984), pp. 612–619.

First FBP works: integral mode acquisitions

- Passive beams[4][5]
- Pencil beams[6]

⇒ No way to sort out individual protons,
⇒ Each pixel actually corresponds to a banana due to multiple Coulomb scattering (cause of the poor spatial resolution[7]),
⇒ Each radiography treated as an x-ray projection, i.e., each pixel measure is assumed to correspond to one straight line.

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[4] K.M. Hanson et al. “Computed tomography using proton energy loss”. In: *Phys Med Biol* 26.6 (1981), pp. 965–983.

[5] P. Zygmanski et al. “The measurement of proton stopping power using proton-cone-beam computed tomography”. In: *Phys Med Biol* 45.2 (2000), pp. 511–528.

[6] Y. Takada et al. “Proton computed tomography with a 250 MeV pulsed beam”. In: *Nucl Instr Meth Phys Res, Sect A* 273.1 (1988), pp. 410–422.

[7] N. Krah et al. “A comprehensive theoretical comparison of proton imaging set-ups in terms of spatial resolution”. In: *Physics in medicine and biology* (2018).

Use of the most likely paths of protons in pCT reconstruction

New scanners can track each proton to estimate their most likely path.

⇒ Reconstruction algorithms using MLP:

- Iterative reconstruction is a natural choice,
- Filtered backprojection, the purpose of this overview.

Binning list-mode data

- FBP from list-mode data has been proposed in other modalities[8].

- It actually comes down to bin after FBP of each event.

⇒ Binning first seems preferable.

[8] J.A. McIntyre. "Computer Assisted Tomography without a Computer". In: *IEEE Transactions on Nuclear Science* 28.1 (1981), pp. 171–173.

Projective solutions

- Use the MLP to optimally chose the corresponding straight line[9],
- Use the MLP to select only those protons that have followed a MLP close to a straight line[10],
- Derive the most likely corresponding projections with straight line paths[11].

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[9] S. Penfold. “Image reconstruction and Monte Carlo simulations in the development of proton computed tomography for applications in proton radiation therapy”. PhD thesis. Centre for Medical Radiation Physics, University of Wollongong, 2010.

[10] G.A.P. Cirrone et al. “Monte Carlo evaluation of the Filtered Back Projection method for image reconstruction in proton computed tomography”. In: *Nucl Instr Meth Phys Res, Sect A* 658.1 (2011), pp. 78–83.

[11] C.A. Collins-Fekete et al. “A maximum likelihood method for high resolution proton radiography/proton CT”. In: *Physics in medicine and biology* 61 (23 2016), pp. 8232–8248.

Distance-driven binning[12]

[12] S. Rit et al. “Filtered backprojection proton CT reconstruction along most likely paths”. In: *Med Phys* 40.3, 031103 (2013), p. 031103.

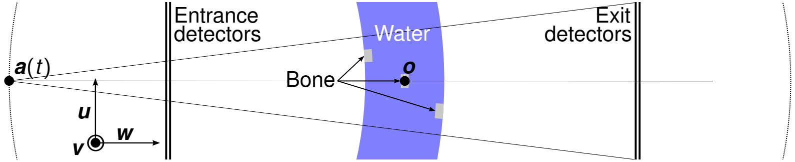
Distance-driven binning

- Named from distance-driven (back)projection[13].
- In practice, binning is computed for several distances between the entrance and the exit detector.

$\Rightarrow$ 4D sinogram $g : \mathbb{R}^3 \times \mathbb{Z} \rightarrow \mathbb{R}$ instead of a standard 3D sinogram, e.g., $g^{out} : \mathbb{R}^2 \times \mathbb{Z} \rightarrow \mathbb{R}$.

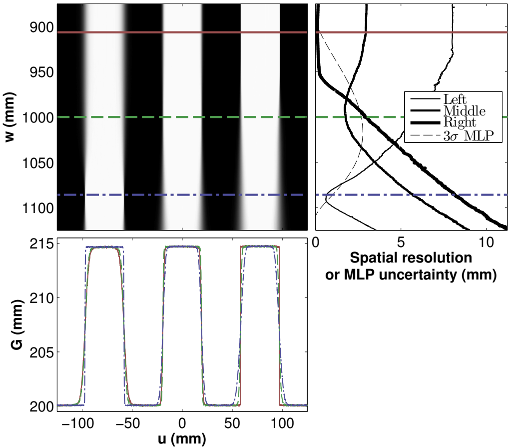
[13] B. De Man and S. Basu. "Distance-driven projection and backprojection in three dimensions". In: *Phys Med Biol* 49.11 (2004), pp. 2463–2475.

Illustration on one projection only[14]



[14] S. Rit et al. "Filtered backprojection proton CT reconstruction along most likely paths". In: *Med Phys* 40.3, 031103 (2013), p. 031103.

Illustration on one projection only[15]



[15] S. Rit et al. "Filtered backprojection proton CT reconstruction along most likely paths". In: *Med Phys* 40.3, 031103 (2013), p. 031103.

Reconstruction algorithm

Adaptation of the FDK algorithm[16] since we chose a cone-beam source on a circular trajectory

- Same 2D processing (weighting and filtering) on each distance of the sinogram
- Rotate and add

$\Rightarrow$ Each proton information is backprojected along its most likely path

- Also adapted to pencil beam acquisitions[17]

Open questions: do we need that much filtering? Could we avoid the rotation which costs an interpolation?

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[16] L.A. Feldkamp, L.C. Davis, and J.W. Kress. "Practical cone-beam algorithm". In: *J Opt Soc Am A* 1.6 (1984), pp. 612–619.

[17] R. Rescigno et al. "A pencil beam approach to proton computed tomography". In: *Medical Physics* 42.11 (2015), pp. 6610–6624.

Backprojection-then-filtering[19]

- Backprojection is the distance-driven binning and the rotation in one step. Addition is done after backprojection of all protons.

- Theoretical issue since backprojection has an infinite support and so does the filter that need to be applied after backprojection.[18]

⇒ Proposes a correction for the DC offset.

- Much less filtering.

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[18] G.T. Gullberg. "The reconstruction of fan-beam data by filtering the back-projection". In: *Computer Graphics and Image Processing* 10.1 (1979), pp. 30–47.

[19] G. Poludniowski, N.M. Allinson, and P.M. Evans. "Proton computed tomography reconstruction using a backprojection-then-filtering approach". In: *Phys Med Biol* 59.24 (2014), pp. 7905–7918.

Backprojection-then-filtering[22]

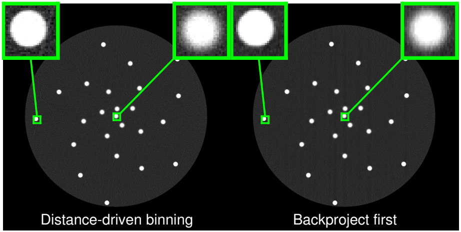
- Based on Noo's two-step Hilbert transform method[20] and Zeng's adaptation to backproject first[21],
- No theoretical difficulty,
- Similar computational complexity,
- Bonus: region-of-interest reconstruction.

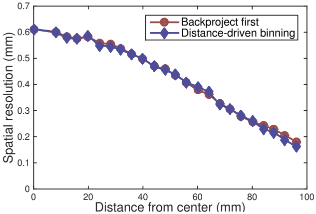
[20] F. Noo, R. Clackdoyle, and J.D. Pack. "A two-step Hilbert transform method for 2D image reconstruction". In: *Phys Med Biol* 49.17 (2004), pp. 3903–3923.

[21] G.L. Zeng. "Image reconstruction via the finite Hilbert transform of the derivative of the backprojection.". In: *Med Phys* 34.7 (2007), pp. 2837–2843.

[22] S. Rit et al. "List-mode proton CT reconstruction using their most likely paths via the finite Hilbert transform of the derivative of the backprojection". In: *Fully 3D Image Reconstruction in Radiology and Nuclear Medicine*. Newport, USA, 2015, pp. 324–327.

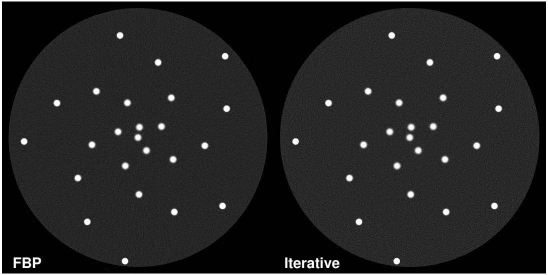
Comparisons[23]

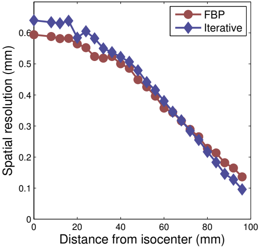




[23] S. Rit et al. "List-mode proton CT reconstruction using their most likely paths via the finite Hilbert transform of the derivative of the backprojection". In: *Fully 3D Image Reconstruction in Radiology and Nuclear Medicine*. Newport, USA, 2015, pp. 324–327.

Comparisons[24][25]





[24] S. Rit et al. “The Reconstruction Toolkit (RTK), an open-source cone-beam CT reconstruction toolkit based on the Insight Toolkit (ITK)”. In: *J. Phys.: Conf. Ser.* 489 (2014), p. 012079.

[25] D.C. Hansen, T. Sangild Sorensen, and S. Rit. “Fast reconstruction of low dose proton CT by sinogram interpolation”. In: *Phys Med Biol* 61.15 (2016), pp. 5868–5882.

Conclusions

- Brief overwiew of FBP algorithms for pCT,
- A few algorithms use the MLP in FBP algorithms,
- Clear differences: computational time, approximations, etc.
- Unclear if the differences in image quality are significant.