A short refresher on filtered backprojection reconstruction for pCT

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2D Radon transform

\[ p(s, \theta) = p_\theta(s) = \int_{\mathbb{R}} f(s \cos \theta - t \sin \theta, s \sin \theta + t \cos \theta) \, dt \]

Figure reprinted from G.L. Zeng. *Medical Image Reconstruction*. Springer, 2010
Noting the 1D and 2D Fourier transforms of $p_\theta$ and $f$

$$P_\theta(w) = \int_{-\infty}^{+\infty} p_\theta(s) e^{-2\pi isw} \, ds$$

$$F(w_x, w_y) = \iint_{\mathbb{R}^2} f(x, y) e^{-2\pi i(xw_x + yw_y)} \, dx \, dy$$

**Theorem**

$$P_\theta(w) = F(w \cos \theta, w \sin \theta)$$
2D inverse Radon transform

\[ f(x, y) = \int_0^\pi \int_{\mathbb{R}} P_\theta(w) |w| e^{2\pi i w (x \cos \theta + y \sin \theta)} \, dw \, d\theta \]

1. Ramp filter the projections,
2. Backprojection of the filtered projections.

Figure reprinted from G.L. Zeng. *Medical Image Reconstruction*. Springer, 2010

From 2D inverse Radon transform to...

... fan-beam CT\(^2\): change of variable.
... FDK algorithm\(^3\): approximate algorithm (except in the central slice) based on 2D fan-beam CT.

In both algorithms:
- same ramp filter,
- additional weighting (before filtering and during backprojection).

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First FBP works: integral mode acquisitions

- Passive beams\(^4\)\(^5\)
- Pencil beams\(^6\)

⇒ No way to sort out individual protons,
⇒ Each pixel actually corresponds to a banana due to multiple Coulomb scattering (cause of the poor spatial resolution\(^7\)),
⇒ Each radiography treated as an x-ray projection, i.e., each pixel measure is assumed to correspond to one straight line.


\(^7\)N. Krah et al. “A comprehensive theoretical comparison of proton imaging set-ups in terms of spatial resolution”. In: *Physics in medicine and biology* (2018).
Use of the most likely paths of protons in pCT reconstruction

New scanners can track each proton to estimate their most likely path.

⇒ Reconstruction algorithms using MLP:
  - Iterative reconstruction is a natural choice,
  - Filtered backprojection, the purpose of this overview.
Binning list-mode data

- FBP from list-mode data has been proposed in other modalities\(^8\).

- It actually comes down to bin after FBP of each event.

⇒ Binning first seems preferable.

Projective solutions

- Use the MLP to optimally chose the corresponding straight line\(^9\),

- Use the MLP to select only those protons that have followed a MLP close to a straight line\(^{10}\),

- Derive the most likely corresponding projections with straight line paths\(^{11}\).

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Distance-driven binning$^{12}$

Distance-driven binning

- Named from distance-driven (back)projection\(^{13}\).

- In practice, binning is computed for several distances between the entrance and the exit detector.

\[ \Rightarrow \text{4D sinogram } g : \mathbb{R}^3 \times \mathbb{Z} \rightarrow \mathbb{R} \text{ instead of a standard 3D sinogram, e.g., } g^{\text{out}} : \mathbb{R}^2 \times \mathbb{Z} \rightarrow \mathbb{R}. \]

Illustration on one projection only\textsuperscript{14}

\begin{itemize}
\item \textbf{Entrance detectors}
\item \textbf{Water}
\item \textbf{Bone}
\item \textbf{Exit detectors}
\end{itemize}

\textsuperscript{14}S. Rit et al. “Filtered backprojection proton CT reconstruction along most likely paths”. In: \textit{Med Phys} 40.3, 031103 (2013), p. 031103.
Illustration on one projection only

Reconstruction algorithm

Adaptation of the FDK algorithm\textsuperscript{16} since we chose a cone-beam source on a circular trajectory

\begin{itemize}
  \item Same 2D processing (weighting and filtering) on each distance of the sinogram
  \item Rotate and add
\end{itemize}

$\Rightarrow$ Each proton information is backprojected along its most likely path

\begin{itemize}
  \item Also adapted to pencil beam acquisitions\textsuperscript{17}
\end{itemize}

Open questions: do we need that much filtering? Could we avoid the rotation which costs an interpolation?

\begin{itemize}
\end{itemize}


\textsuperscript{17}R. Rescigno et al. “A pencil beam approach to proton computed tomography”. In: \textit{Medical Physics} 42.11 (2015), pp. 6610–6624.
Backprojection-then-filtering

- Backprojection is the distance-driven binning and the rotation in one step. Addition is done after backprojection of all protons.

- Theoretical issue since backprojection has an infinite support and so does the filter that need to be applied after backprojection.\(^\text{18}\)

\[\implies\] Proposes a correction for the DC offset.

- Much less filtering.


Backprojection-then-filtering\textsuperscript{22}

- Based on Noo’s two-step Hilbert transform method\textsuperscript{20} and Zeng’s adaptation to backproject first\textsuperscript{21},

- No theoretical difficulty,

- Similar computational complexity,

- Bonus: region-of-interest reconstruction.


\textsuperscript{22} S. Rit et al. “List-mode proton CT reconstruction using their most likely paths via the finite Hilbert transform of the derivative of the backprojection”. In: \textit{Fully 3D Image Reconstruction in Radiology and Nuclear Medicine}. Newport, USA, 2015, pp. 324–327.
Comparisons

S. Rit et al. “List-mode proton CT reconstruction using their most likely paths via the finite Hilbert transform of the derivative of the backprojection”. In: *Fully 3D Image Reconstruction in Radiology and Nuclear Medicine*. Newport, USA, 2015, pp. 324–327.
Comparisons

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Conclusions

- Brief overview of FBP algorithms for pCT,
- A few algorithms use the MLP in FBP algorithms,
- Clear differences: computational time, approximations, etc.
- Unclear if the differences in image quality are significant.