# A deconvolution method to improve spatial resolution in proton CT

6th Annual Loma Linda Workshop

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# Context

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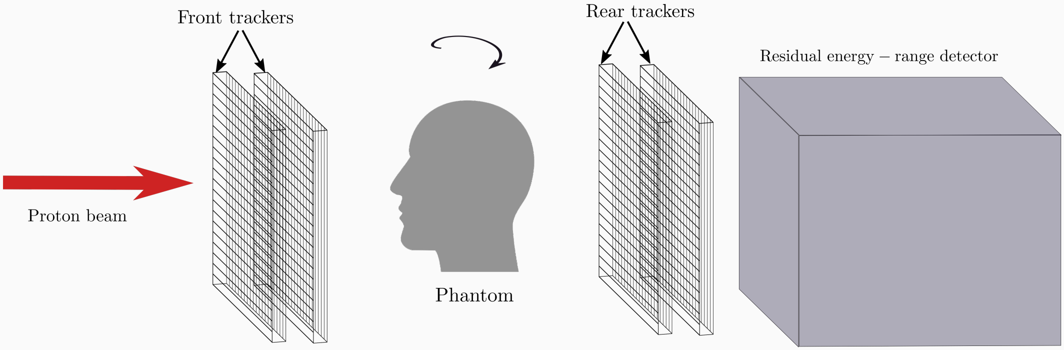
- List-mode set-up

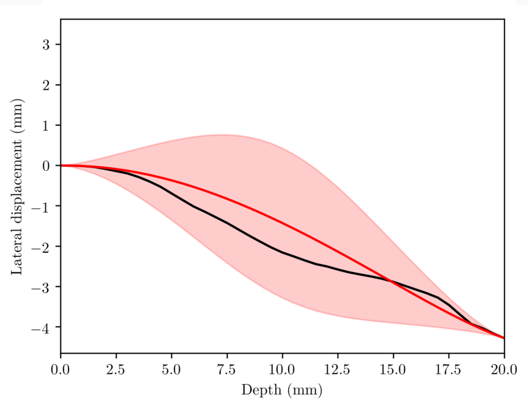


# Context

- The inverse problem in energy loss proton CT is

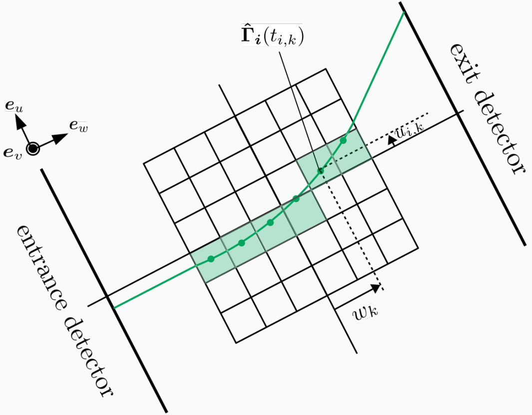
$$\int_{\hat{\Gamma}_i} \mathrm{RSP}(\boldsymbol{x})\mathrm{d}l = \mathrm{WEPL}_i, \qquad (1)$$

  where $\hat{\Gamma}_i$ is the $i$-th proton path, and $\mathrm{WEPL}$ is the water equivalent path length.
- Proton path is approximated using the most likely path formalism based on [Schulte et al., 2008]



# Distance-driven binning

- Distance-driven binning algorithm [Rit et al., 2013]




- Discretize space, pixel index $j$
- Discretize direction, source position index $p$
- Sample MLP, depth index $k$

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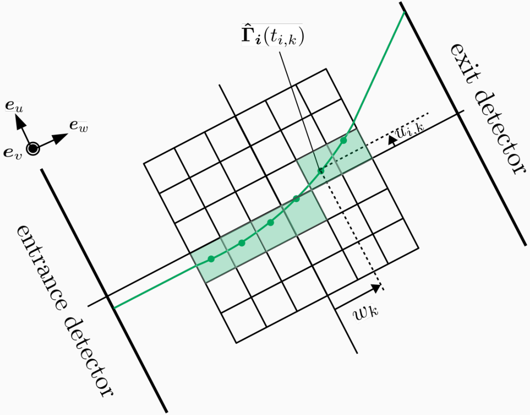
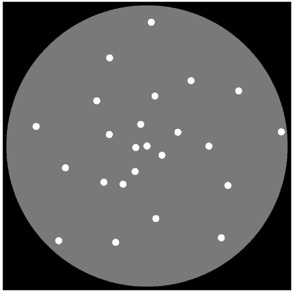



- Discretize space, pixel index $j$
- Discretize direction, source position index $p$
- Sample MLP, depth index $k$
- Average WEPL of protons from source position $p$ going through pixel $j$

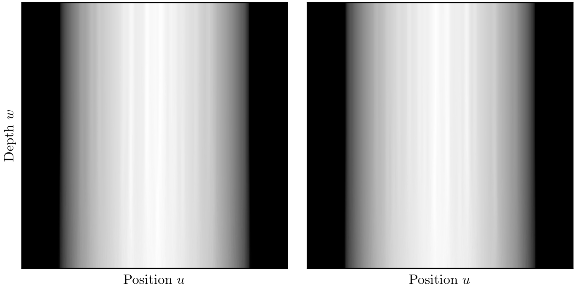
$$g_{j,p} = \frac{\sum_{i \in \mathbb{I}_p} \sum_k \zeta_j(u_{i,k}, v_{i,k}, w_k) \mathrm{WEPL}_i}{\sum_{i \in \mathbb{I}_p} \sum_k \zeta_j(u_{i,k}, v_{i,k}, w_k)} \tag{2}$$

with $\zeta_j$ the indicator function for pixel $j$.

# Distance-driven binning

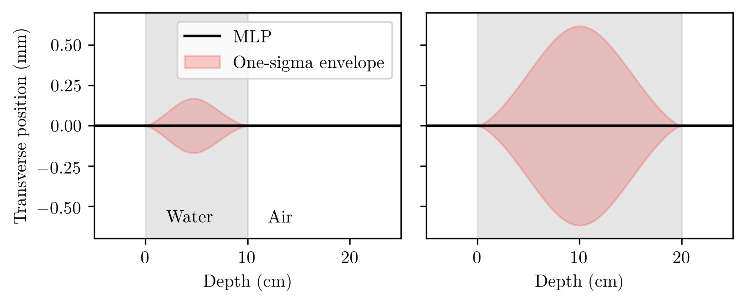


(a) Spiral phantom




(b) Distance-driven projections at angles 0° and 90°

# Objective




- Is it possible to deblur the distance-driven projections using the MLP uncertainty?

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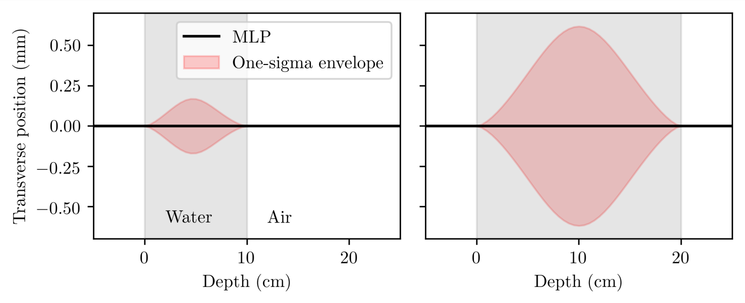


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- Blur in a pixel of the reconstructed image is due to a combination of the uncertainty of different protons coming from different angles $\rightarrow$ deconvolution in projection space

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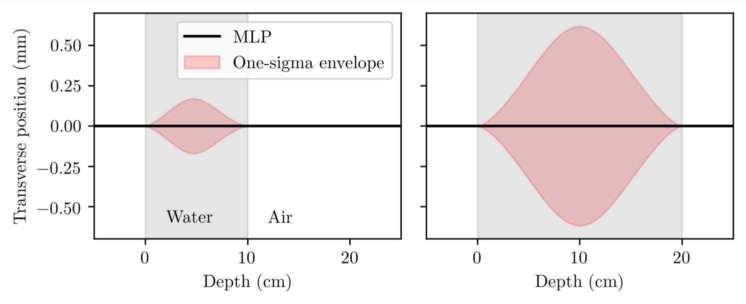


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- Blur in a pixel of the reconstructed image is due to a combination of the uncertainty of different protons coming from different angles → deconvolution in projection space
- Uncertainty in the binned data depends on depth inside object, projection angle and transverse position

# Methods

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- Gaussian blurring kernel

$$h[u - u', \sigma_p(u', w)] \propto \exp\Big( - \frac{(u - u')^2}{2\sigma_p(u', w)^2} \Big). \tag{4}$$

where $\sigma_p(u', w)$ represents the MLP uncertainty of protons with source position $p$ at depth $w$ and position $u'$.

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  where $\sigma_p(u', w)$ represents the MLP uncertainty of protons with source position $p$ at depth $w$ and position $u'$.
- Distance-driven binning of MLP uncertainty

$$\sigma_{j,p} = \sqrt{\frac{\sum_{i\in\mathbb{I}_p}\sum_k \zeta_j(u_{i,k}, v_{i,k}, w_k)\sigma^2_{\mathrm{MLP},i}(w_k)}{\sum_{i\in\mathbb{I}_p}\sum_k \zeta_j(u_{i,k}, v_{i,k}, w_k)}} \tag{5}$$

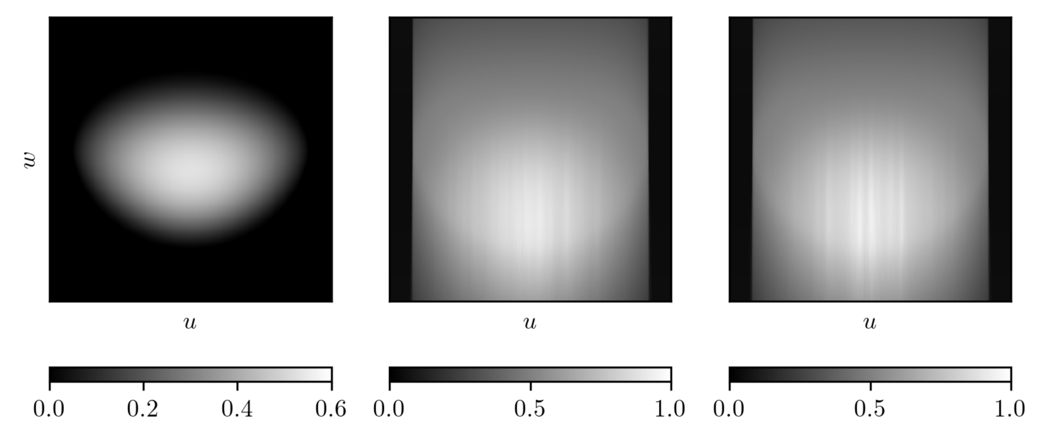
# MLP uncertainty

- Standard MLP formalism [Schulte et al., 2008]
- Extended MLP formalism [Krah et al., 2018] to take into account tracker spatial and angular resolution
  - Spatial resolution $\sigma_t = 0.066$ mm
  - Material budget $x/X_0 = 5 \times 10^{-3}$
  - Distance between trackers $d_T = 10$ cm
  - Distance trackers-isocenter $30 - 40$ cm

# Uncertainty maps: spiral phantom

Standard formalism (ideal trackers)

Extended formalism (realistic trackers) at 0° and 90°

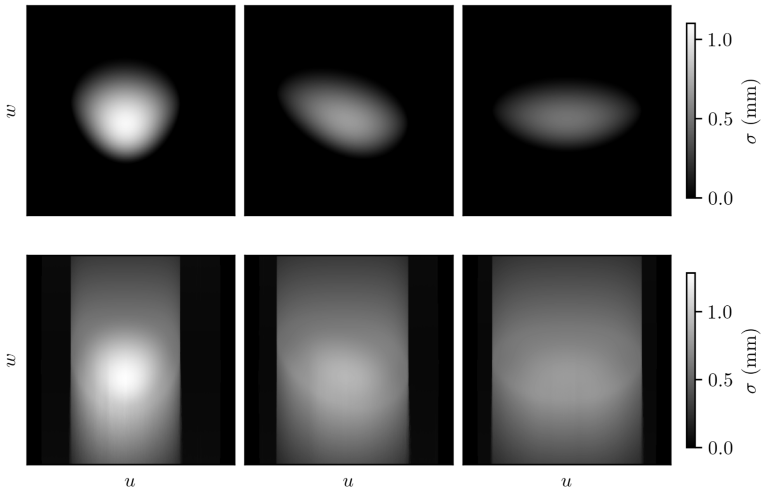


# Uncertainty maps: pelvis phantom

0° 45° 90°

Standard formalism
(ideal trackers)

Extended formalism
(realistic trackers)



# Shift-variant deconvolution

- $$g_p(u,w) = \int h[u-u', \sigma_p(u',w)]\hat{g}_p(u',w)\,\mathrm{d}u' \tag{6}$$
- In matrix notation, we have
$$\mathbf{g}_w = \mathbf{H}_w \hat{\mathbf{g}}_w \tag{7}$$
where $\mathbf{g}_w$ is a vector containing one distance $w$ of the distance-driven projection, $\mathbf{H}_w$ is the shift-variant convolution matrix for this distance.
- The solution is given by
$$\hat{\mathbf{g}}_w = \mathbf{H}_w^{-1} \mathbf{g}_w. \tag{8}$$

# Truncated SVD

- Singular value decomposition (SVD):

$$\mathbf{H}_w = \mathbf{U}\mathbf{S}\mathbf{V}^T \tag{9}$$

where $\mathbf{U} = [\mathbf{u}_1, ..., \mathbf{u}_N]$ and $\mathbf{V} = [\mathbf{v}_1, ..., \mathbf{v}_N]$ are orthogonal matrices; and $\mathbf{S} = \mathrm{diag}(s_1, ..., s_N)$ is a diagonal matrix.

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$$\hat{\mathbf{g}}_w = \sum_{i=1}^{N} \frac{\mathbf{u}_i^T \mathbf{g}_w}{s_i} \mathbf{v}_i. \tag{10}$$

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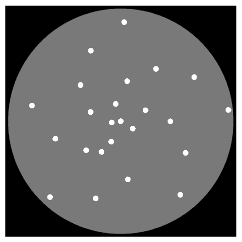
$$\hat{\mathbf{g}}_w = \sum_{i=1}^{N} \frac{\mathbf{u}_i^T \mathbf{g}_w}{s_i} \mathbf{v}_i. \tag{10}$$

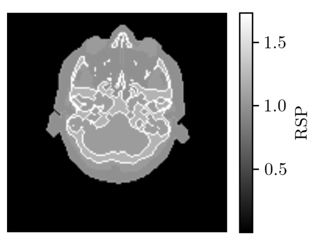
- Truncated SVD: we choose a cut-off value $N_c < N$

$$\hat{\mathbf{g}}_w = \sum_{i=1}^{N_c} \frac{\mathbf{u}_i^T \mathbf{g}_w}{s_i} \mathbf{v}_i. \tag{11}$$

# Simulations

- Monte Carlo simulations with Gate [Jan et al., 2011]
- Spiral phantom + head ICRP phantom
- 200/250 MeV fan beam over 360°
- Data acquired using either ideal or realistic trackers
- Spatial resolution measured using frequency corresponding to 10% of the MTF's peak value





# Results

# Results: spiral phantom

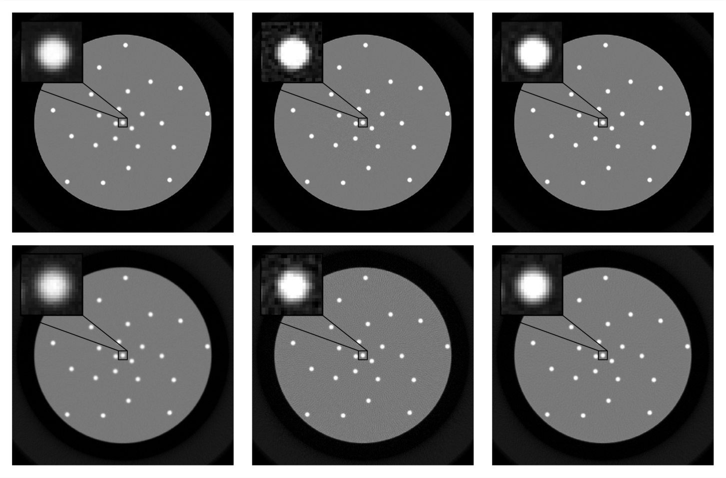
No deconvolution

Deconvolution w/ truncation at 10%

Deconvolution w/ truncation at 30%

Ideal trackers

Realistic trackers

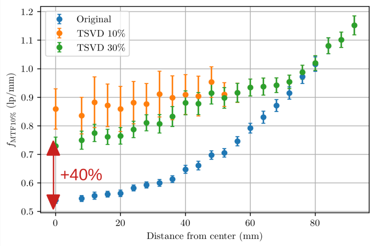


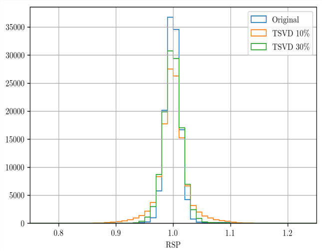
# Results: spiral phantom

Spatial resolution as a function of distance

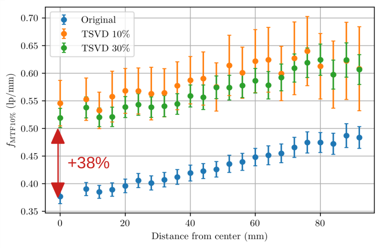
RSP distribution in water region

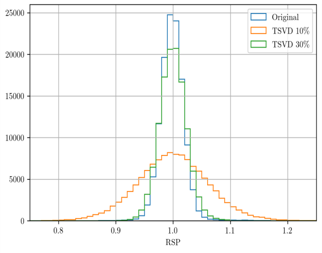
Ideal trackers



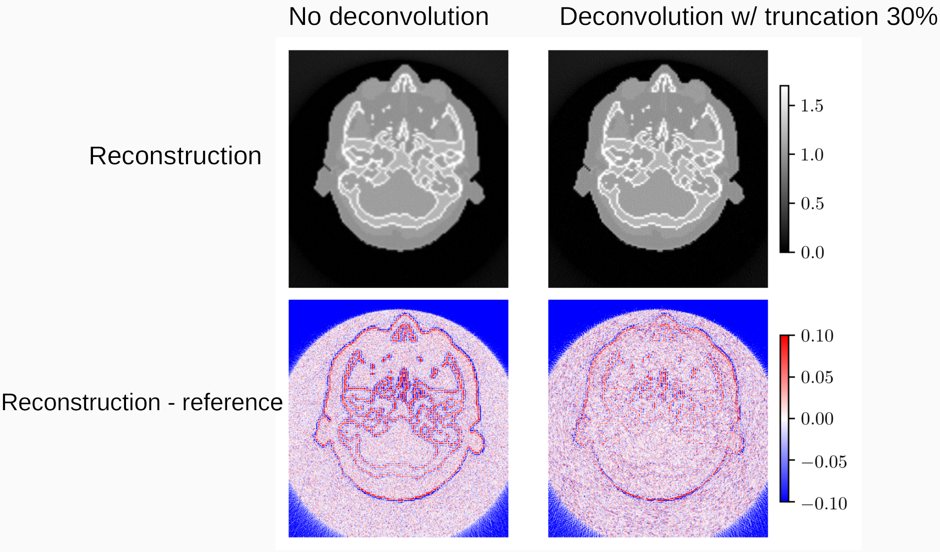




Realistic trackers



# Results: head phantom w/ realistic trackers



# Conclusions

- Spatial resolution can be improved by using the MLP uncertainty
- The resolution in a spiral phantom was increased by up to $38\%$ for realistic trackers and $40\%$ for ideal trackers while keeping a similar noise level as in the reconstruction without deconvolution
- The choice of the truncation level of the SVD should be better adapted to the noise level in the projections
- High computational cost but parallelizable

Jan, S., Benoit, D., Becheva, E., Carlier, T., Cassol, F., Descourt, P., Frisson, T., Grevillot, L., Guigues, L., Maigne, L., et al. (2011).
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Rit, S., Dedes, G., Freud, N., Sarrut, D., and Létang, J. M. (2013).
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