Scattering proton CT

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Based on PhD thesis of CQ; paper under review in PMB

Loma Linda workshop 2020

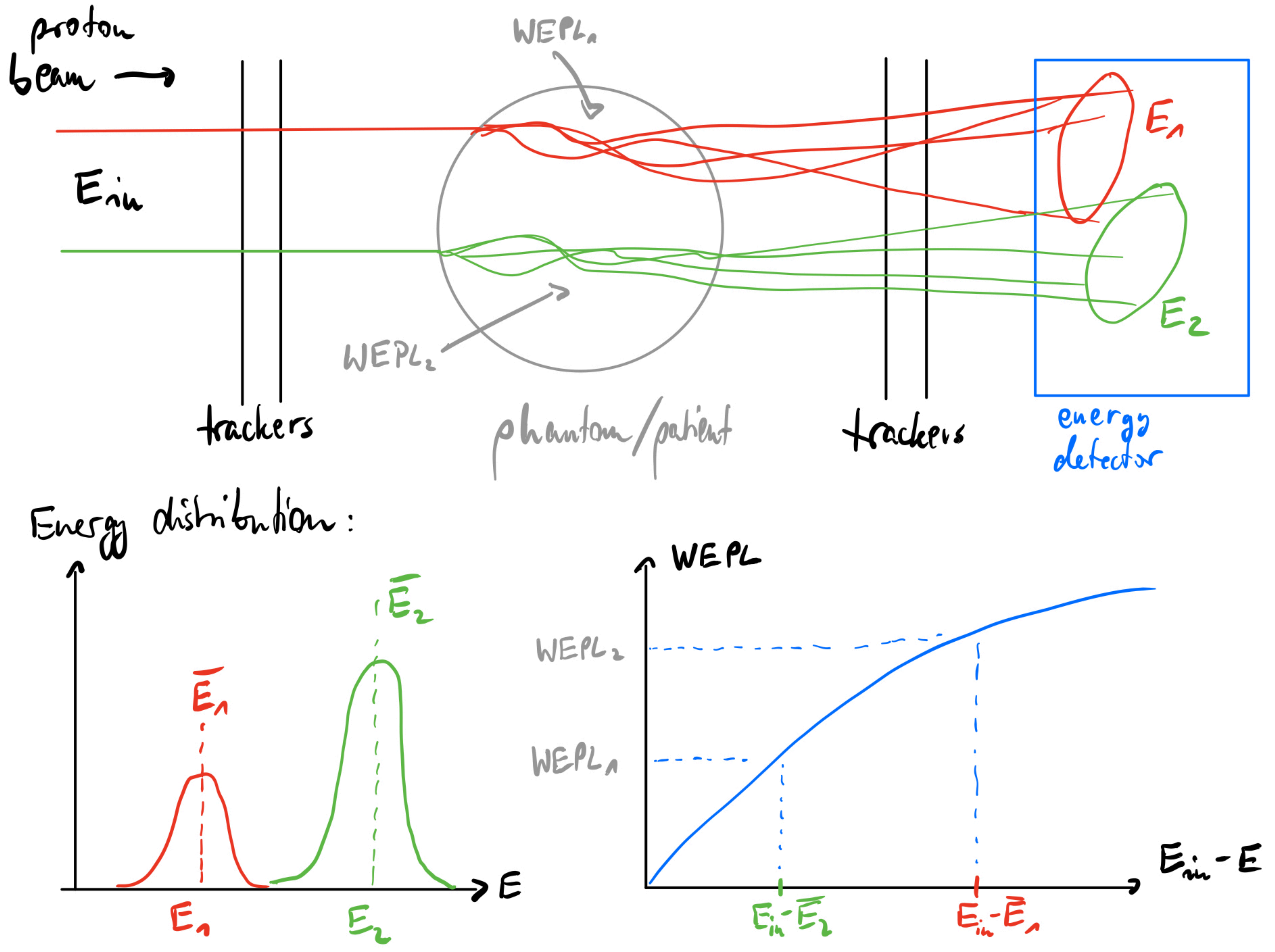
I will speak about …

- Rationale behind scattering proton CT
- Relative scattering power
- How to reconstruct scattering images
- Statistical limitation of scattering proton CT

Rationale

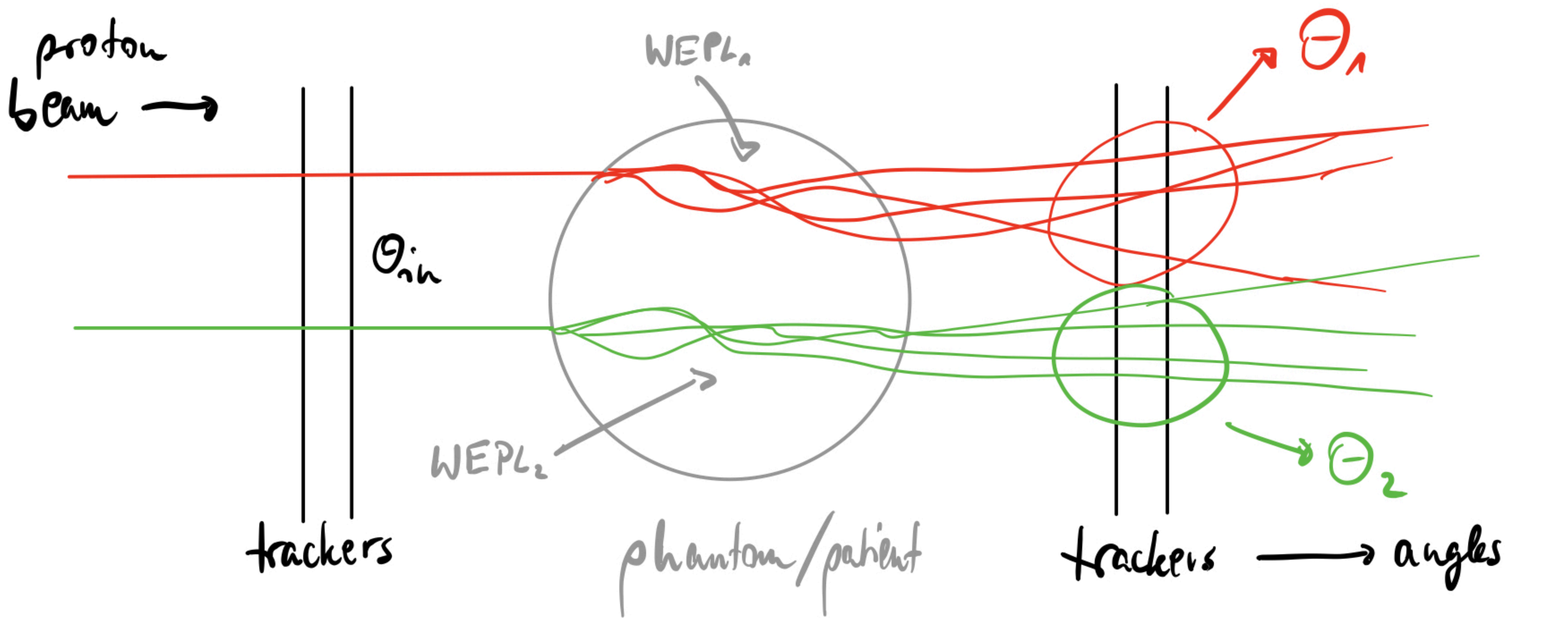
Recall:

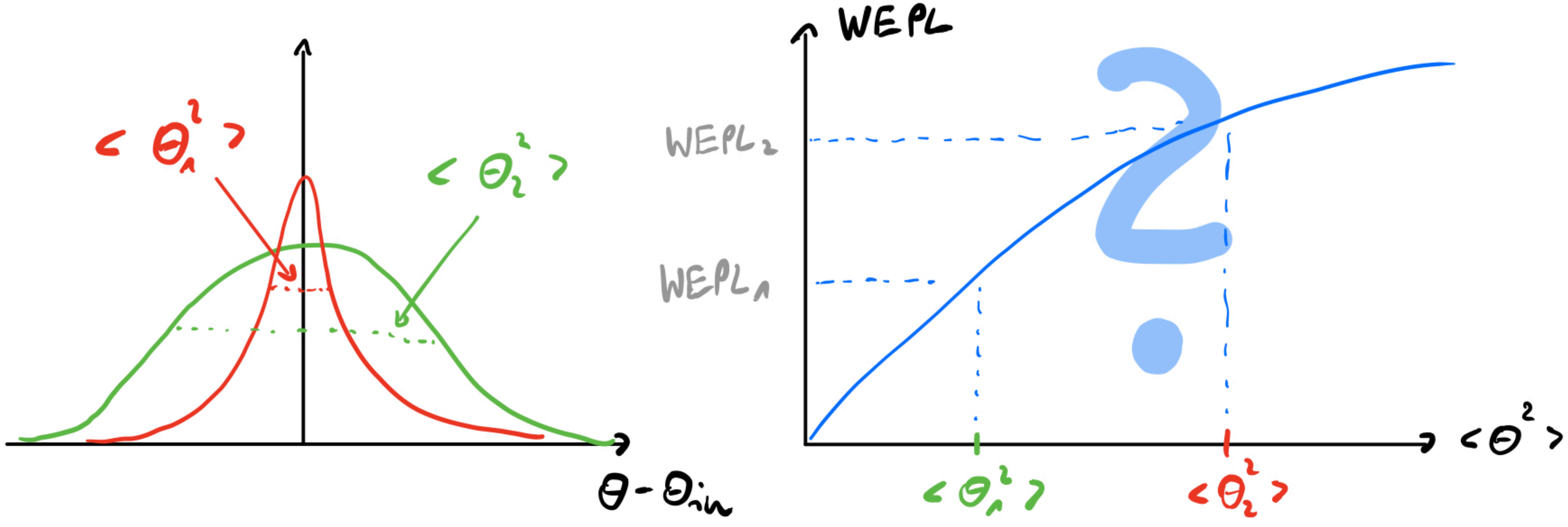
Energy-loss proton CT



Rationale

Remove energy detector and look at angles instead …





Tomographic problem of scattering pCT

Short hand notation for variance: $A \equiv \langle\theta\rangle^2$

Scattering power:

$$T(z) = \frac{\mathrm{d}A(z)}{\mathrm{d}z} \quad \Leftrightarrow A(z) = \int_0^z T(z')\mathrm{d}z' + A_0$$

Tomographic reconstruction

Radiation length X_0: local quantity

Energy E: depends on traversed material

Clearer from model [1]: $A(z) \equiv \langle\theta\rangle^2(z) = \left(1 + \frac{1}{9}\log_{10}\frac{z}{X_0}\right)^2 \Omega_0^2 \int_0^z \frac{(E(z') + E_p)^2 c^2}{(E(z') + 2E_p)^2 E^2(z')} \frac{1}{X_0(z')}\mathrm{d}z'$

[1] Highland, V. L. (1975). Some practical remarks on multiple scattering. Nuclear Instruments and Methods, 129(2), 497–499.

Tomographic problem of scattering pCT

Two options:

1. Estimate energy term from a priori knowledge and incorporate into reconstruction. Example: Bopp 2013.

2. Define a new quantity which depends only weakly on energy: Our approach and used in Taylor 2016.

→ **Relative scattering power**

For electrons:

Yesterday's talk by Daiki and

Jansen, H., & Schütze, P. (2018). Feasibility of track-based multiple scattering tomography. Applied Physics Letters, 112(14), 144101.

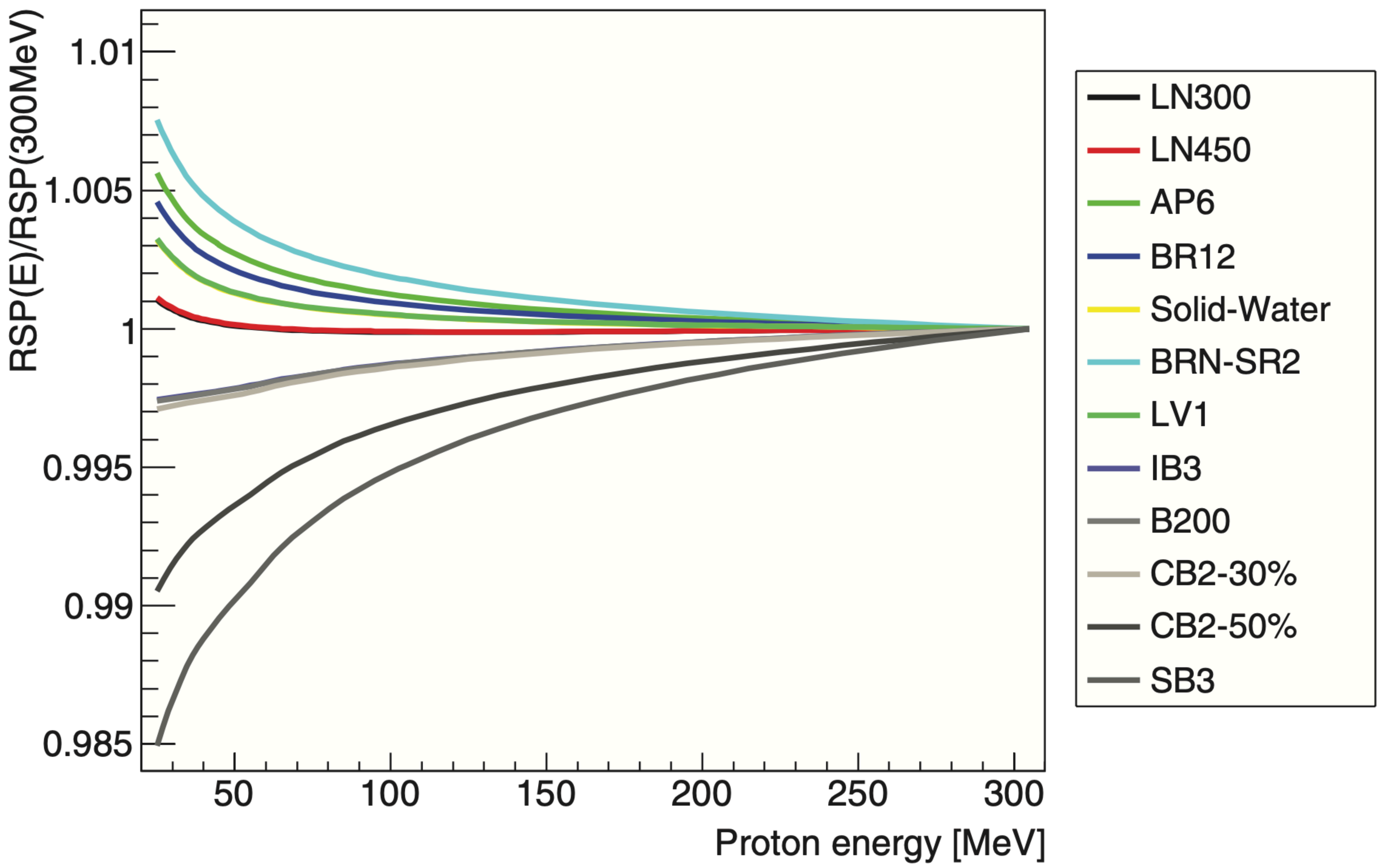
Recall: Relative stopping power

- Proton stopping power also depends on energy, and strongly so:

$$S \equiv \frac{\mathrm{d}E}{\mathrm{d}z} \propto \frac{1}{E^b} \quad \text{with } b \approx 1.7$$

- But stopping power **relative** to water is almost independent of energy.

$$\mathrm{RSP} \equiv \frac{S}{S_w}$$



Arbor, N., …, Rit, S. (2015). Monte Carlo comparison of x-ray and proton CT for range calculations of proton therapy beams. Physics in Medicine and Biology, 60(19), 7585.

Relative scattering power

- There is a one-to-one mapping *h* between energy E and angular variance A, so that $E = h(A)$

- … and we can define the scattering power as a function of angular variance:

$$\tau(X_0(z), A(z)) \equiv T(X_0(z), h(A(z)))$$

- … and the relative scattering power as

$$\delta(X_0(z), A(z)) = \frac{\tau(X_0(z), A(z))}{\tau_w(A)}$$

- The tomographic integration problem is now

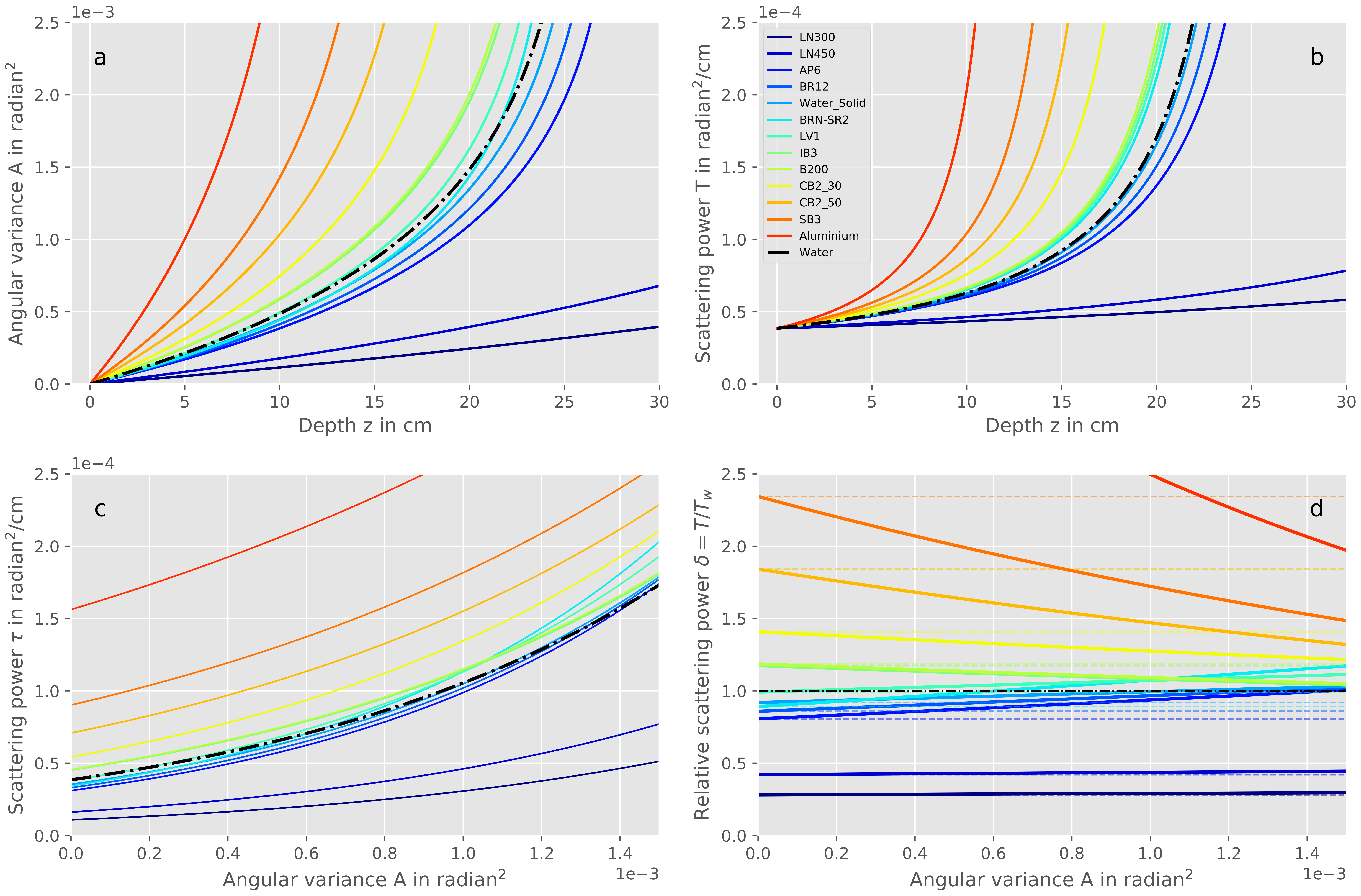
$$G(A_{\text{out}}) = \int_0^{A_{\text{out}}} \frac{1}{T_w(A)} \mathrm{d}A = \int_0^{z_{\text{out}}} \delta(X_0(z), A(z)) \mathrm{d}z$$

Water equivalent scattering path length

Reconstructed quantity

How much does δ depend on A?

How much does δ depend on A?



200 MeV protons

Dashed lines: Ratio of radiation lengths

$$X_{0,w}/X_0$$

Distance driven binning - reconstruction

- Estimate angular variance in pixel j and depth w from all protons in a projection whose MLP crosses the pixel:

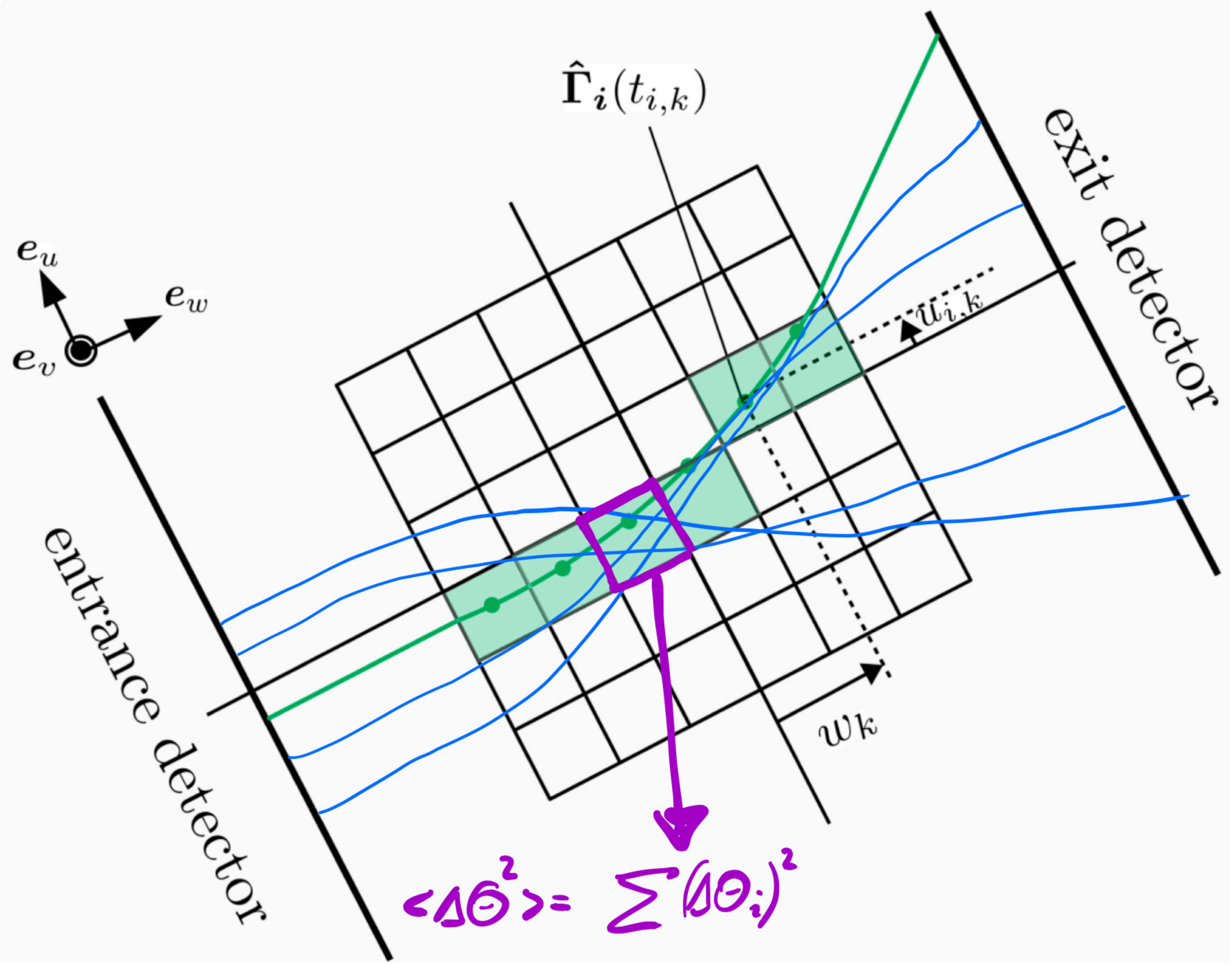
$$\tilde{A}_j(w) = \frac{\sum_{i \in I} h_j(u_i(w), v_i(w))[(\Delta\theta_i^u)^2 + (\Delta\theta_i^v)^2]/2}{\sum_{i \in I} h_j(u_i(w), v_i(w))}$$

with h_j the pixel indicator function.

- Convert to water equivalent scattering length *g* using a pre-built LUT:

$$g_j(w) = G_{\mathrm{LUT}}(\tilde{A}_j(w))$$

- Reconstruct relative scattering power from g_j using FDK algorithm[1]

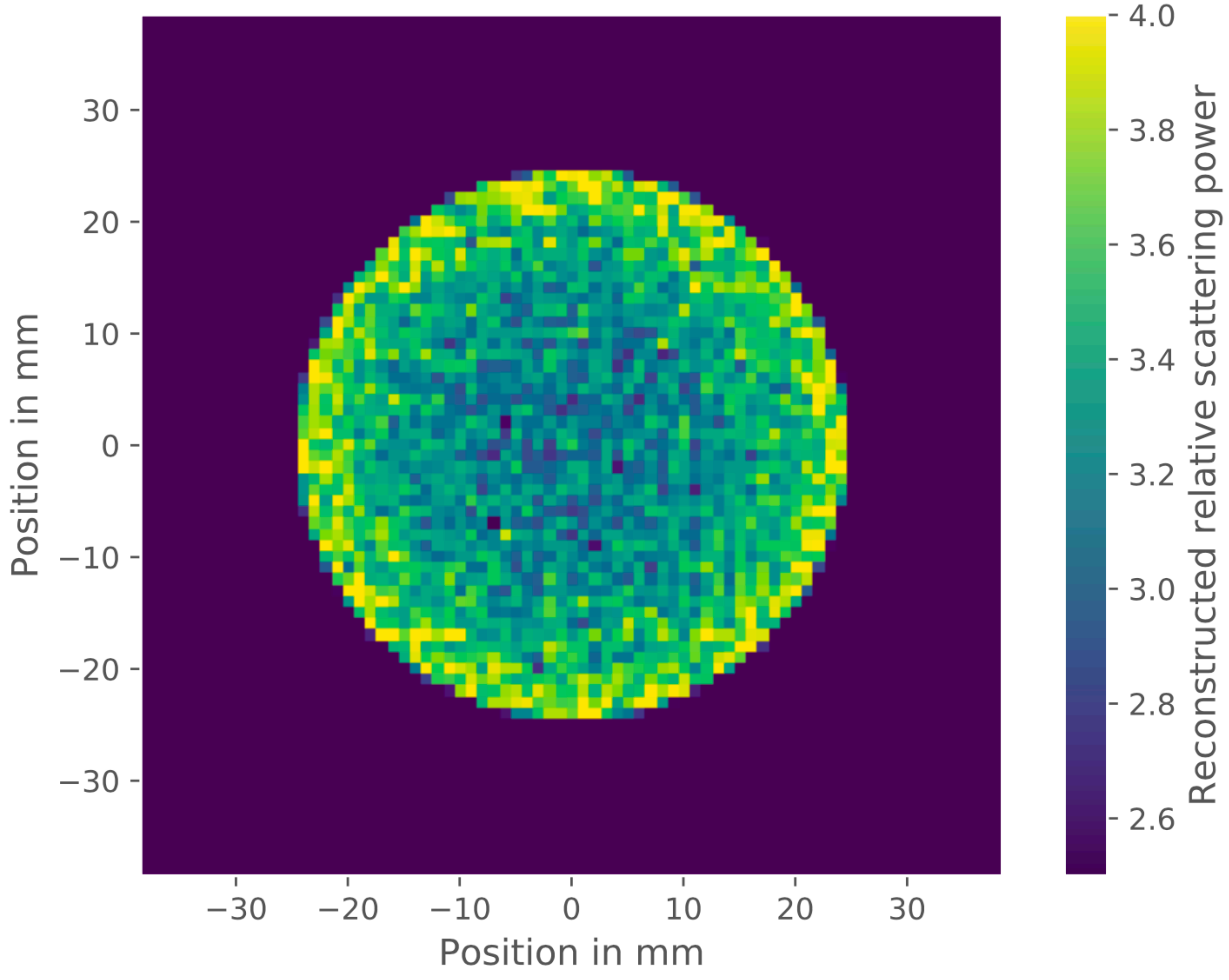


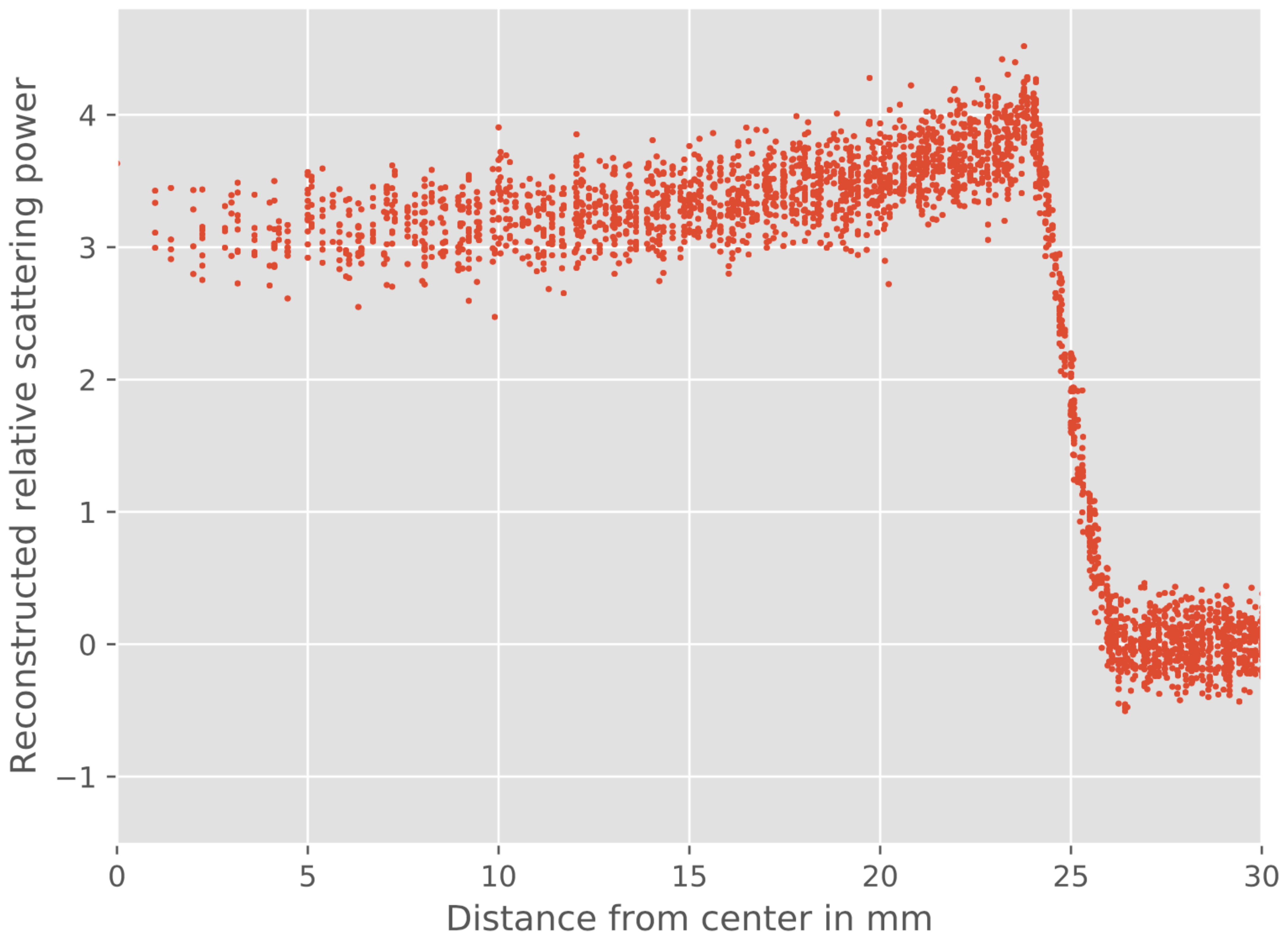
Adapted from Feriel Khellaf's talk earlier today

[1] Rit, S., Dedes, G., Freud, N., Sarrut, D., & Létang, J. M. (2013). Filtered backprojection proton CT reconstruction along most likely paths. Medical Physics, 40(3), 031103

Cupping artifact in dense object

- Reconstructed relative scattering power map of an aluminum cylinder
  based on Monte Carlo simulated data (GATE)



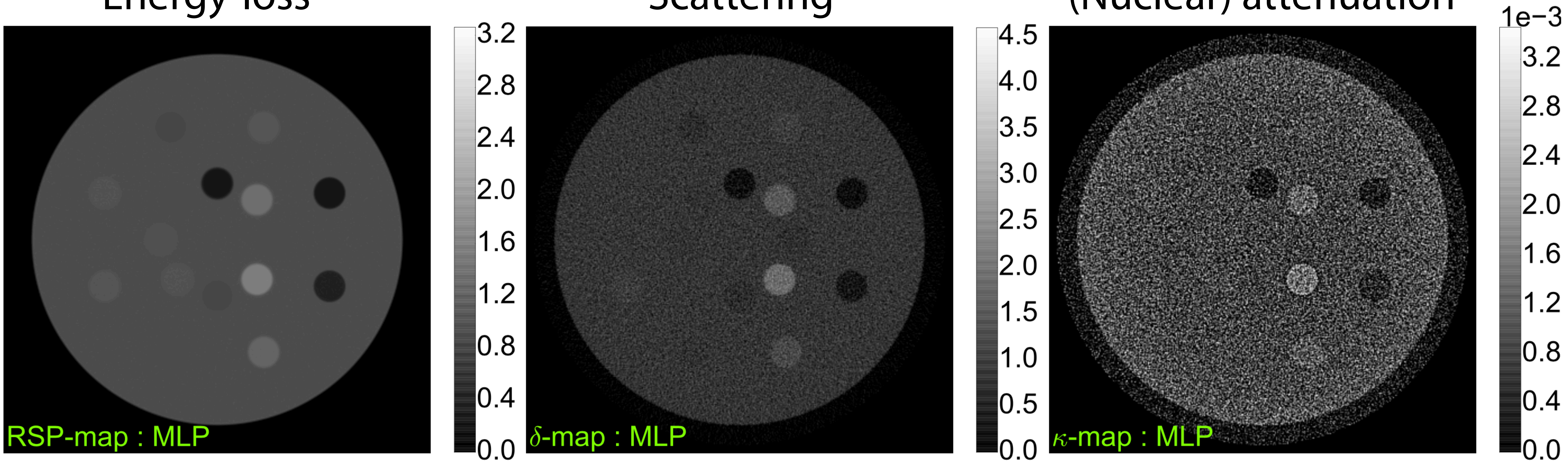


Cupping artifact because δ depends on A
(Al is denser than water)

Gammex phantom

- Reconstructed relative scattering power map of Gammex phantom
  based on Monte Carlo simulated data (GATE)

Energy-loss | Scattering | (Nuclear) attenuation[1]

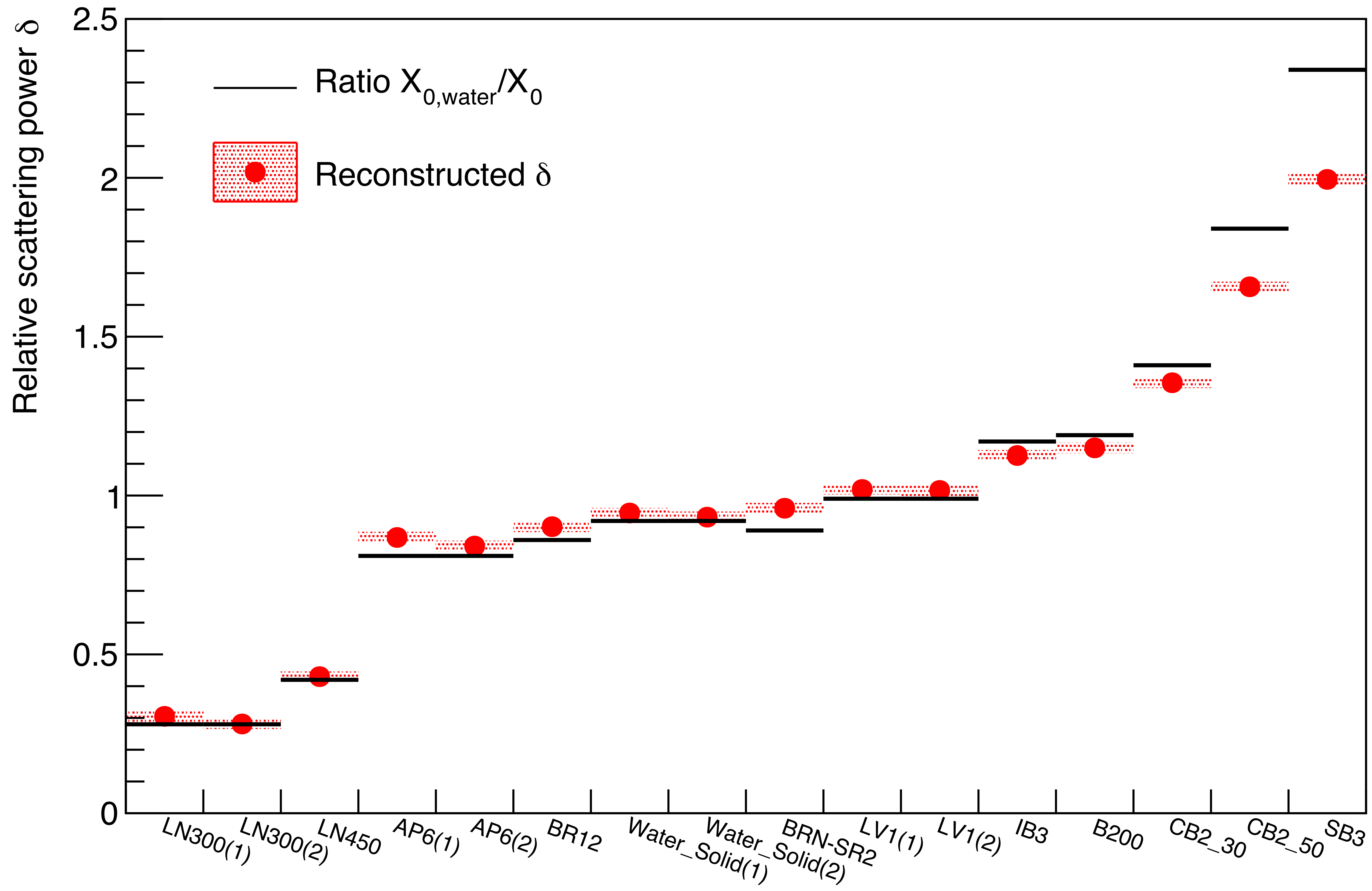


→ Scattering pCT noisier than energy-loss

[1] Quiñones, C. T., Létang, J. M., & Rit, S. (2016). Filtered back-projection reconstruction for attenuation proton CT along most likely paths. Physics in Medicine and Biology, 61(9), 3258–3278.

Gammex phantom: Accuracy

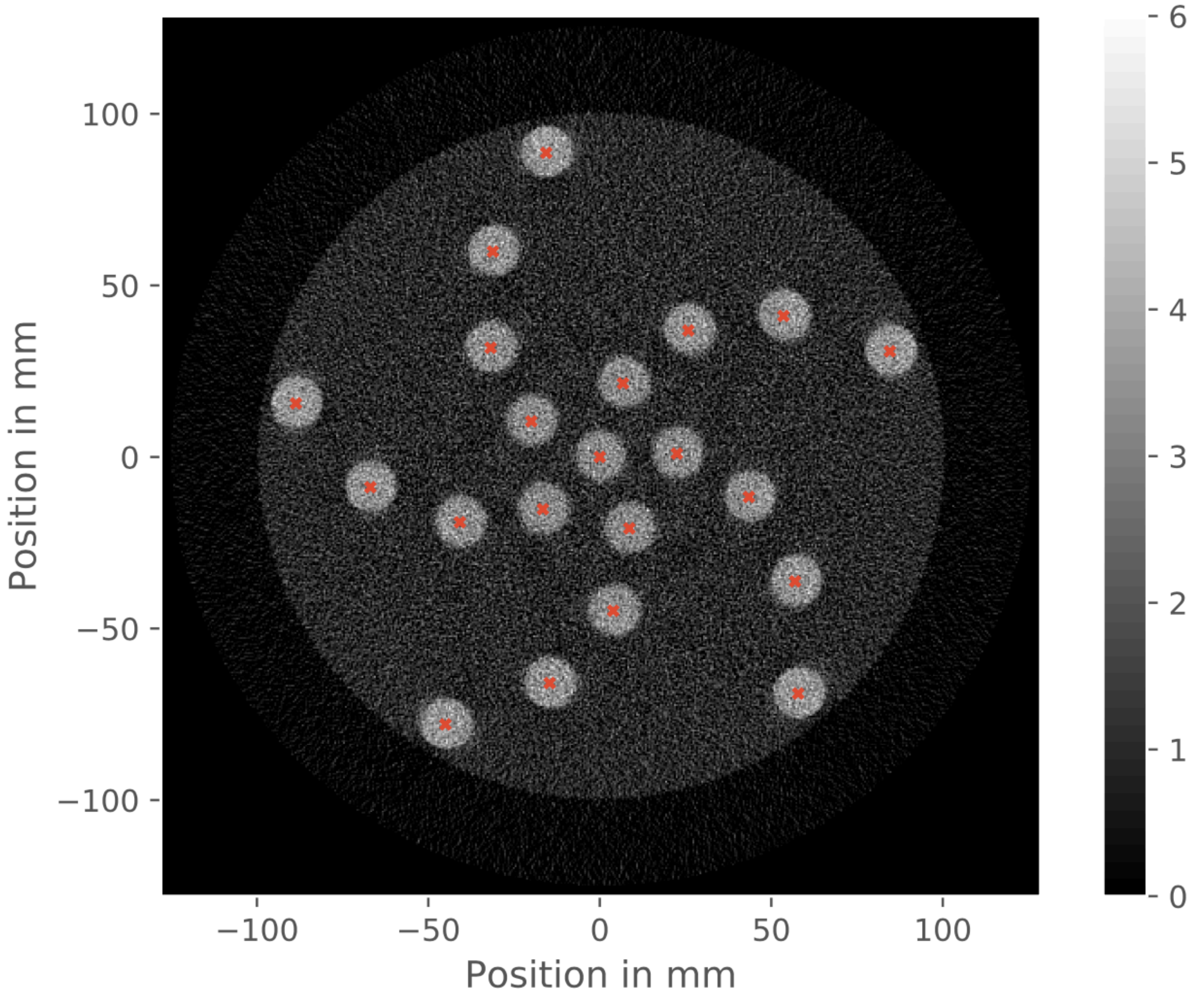
- Relative scattering power values compared with ratio of radiation length $X_{0,w}/X_0$

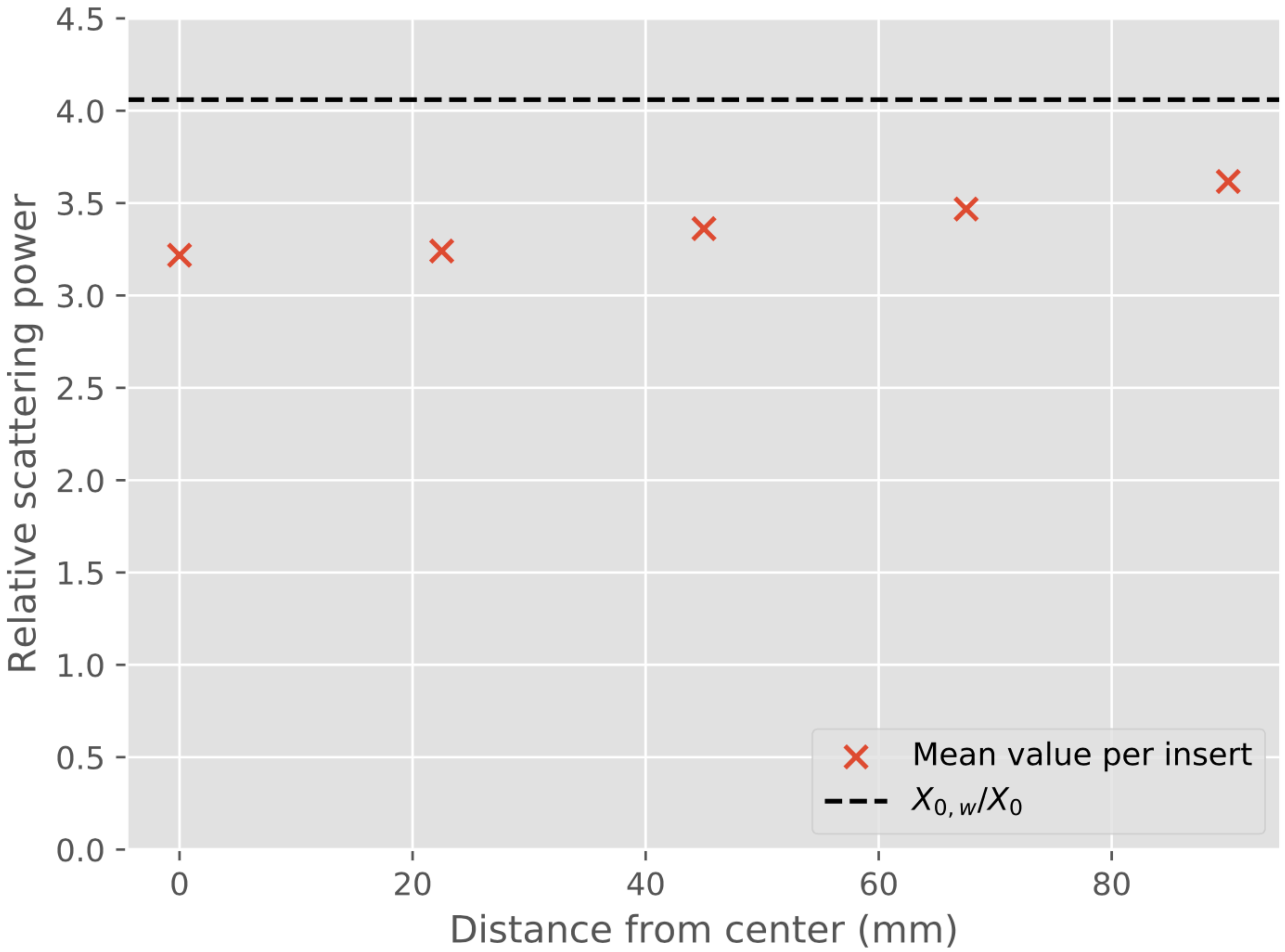


Less dense than water ⟷ Denser than water

Spiral phantom with aluminum inserts

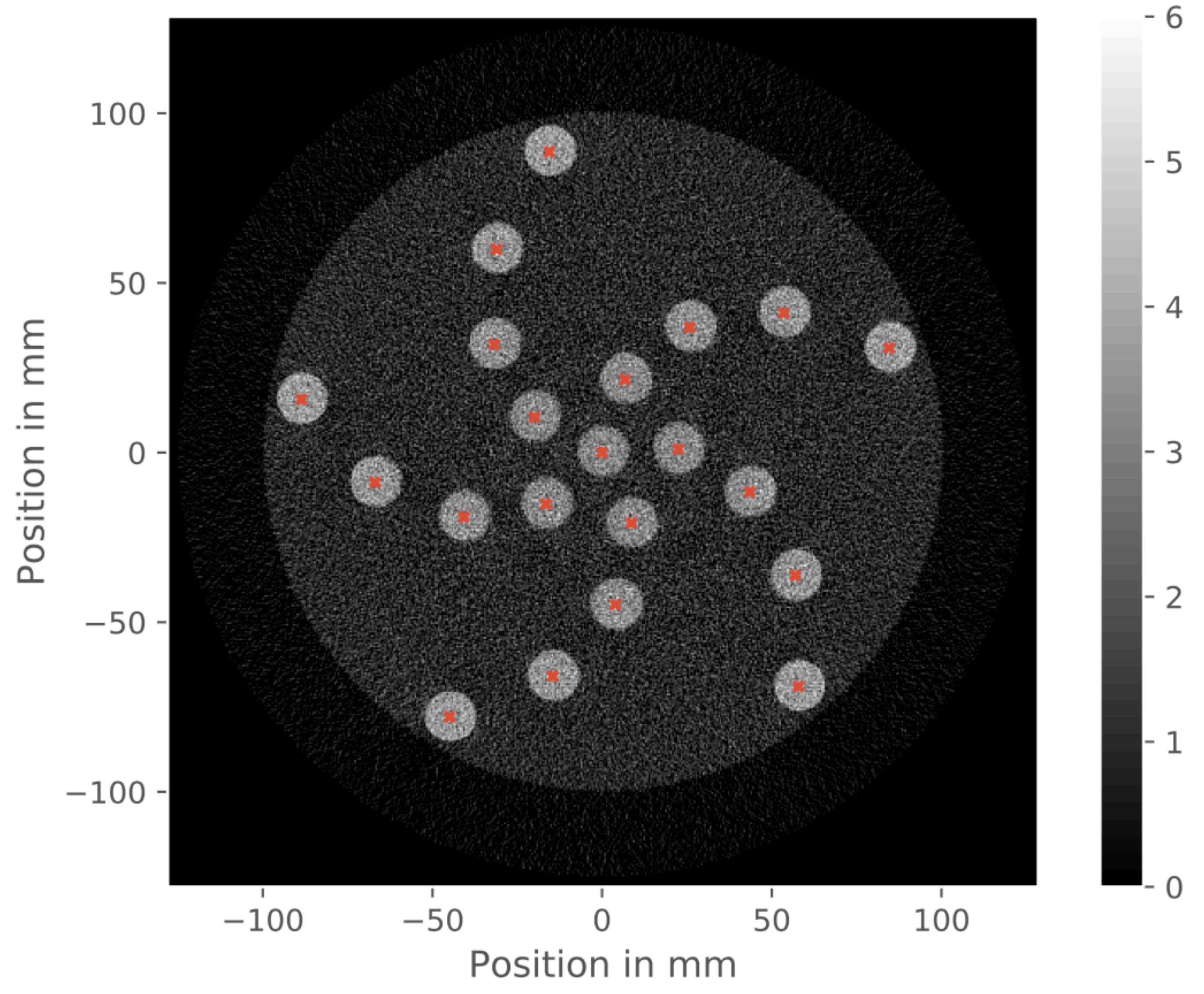
- Relative scattering power values compared with ratio of radiation length $X_{0,w}/X_0$ based on Monte Carlo simulated data (GATE)

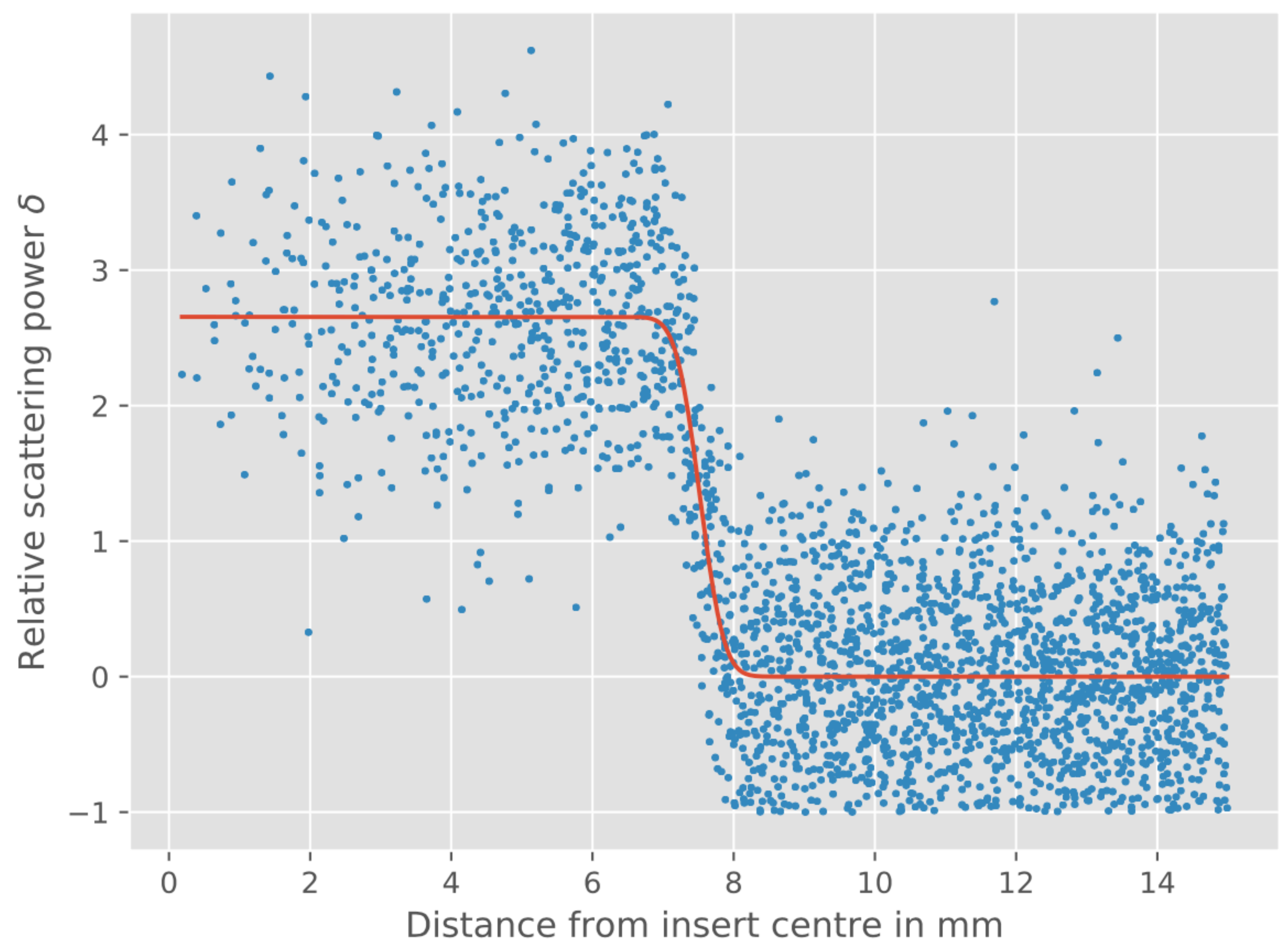




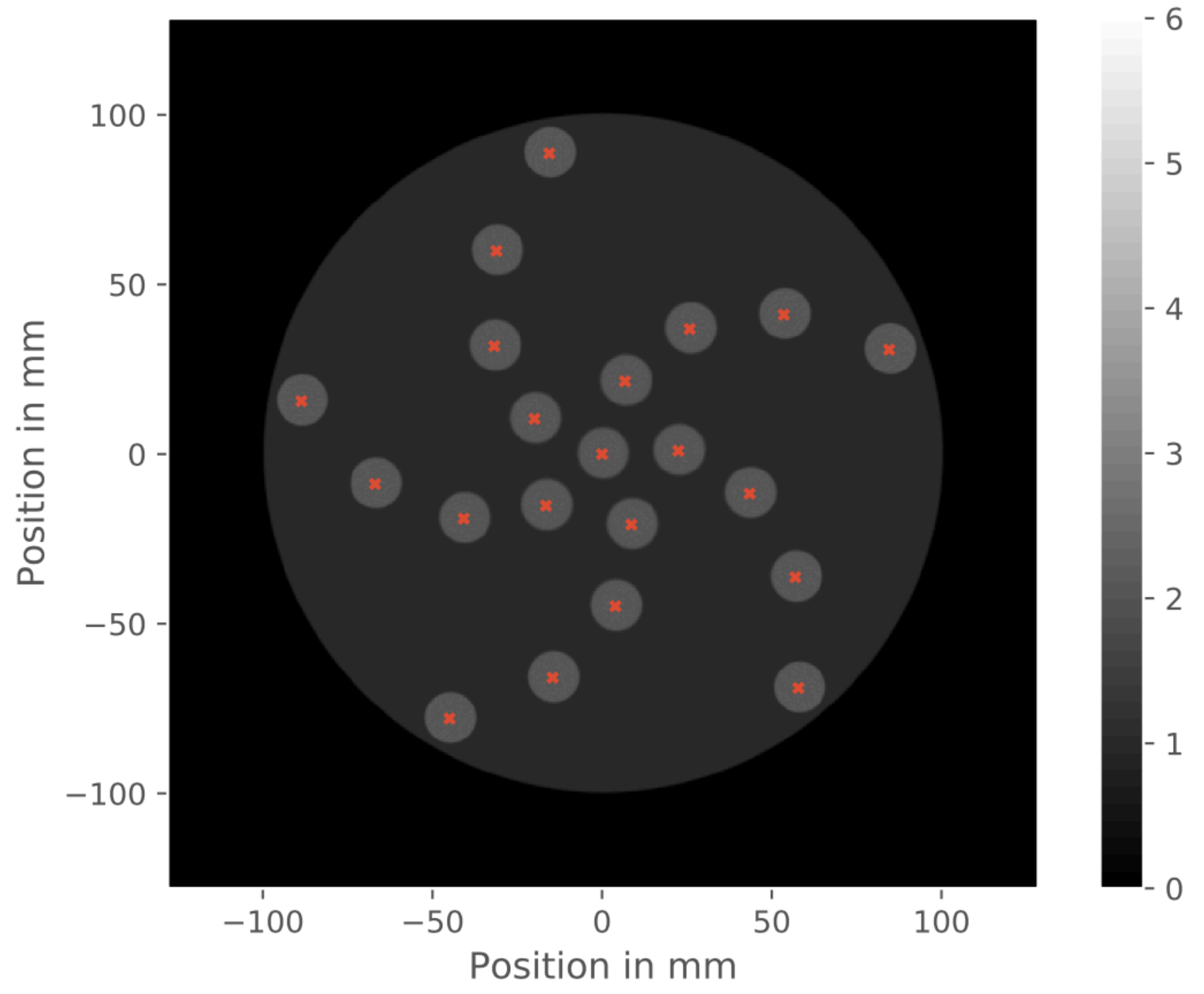
Again slight cupping artifact

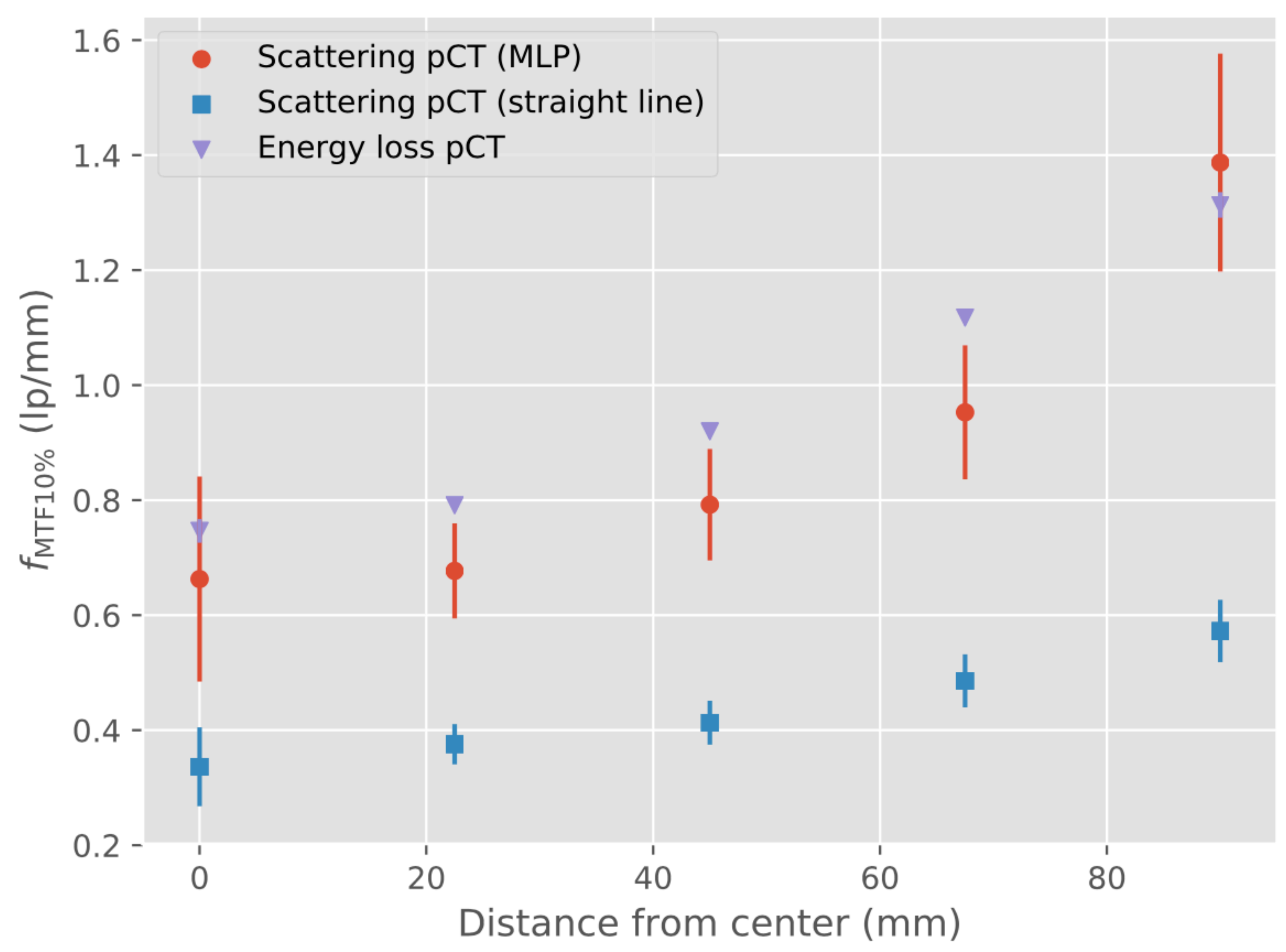
Spiral phantom with aluminum inserts





Spatial resolution estimated from edge spread function





Spatial resolution comparable with energy-loss proton CT

Statistical limitations of scattering proton CT

… calculated at the center of a water cylinder

- Variance is estimated from finite number of samples during distance driven binning:

$$\tilde{A} = \frac{1}{2N} \sum_{i \in I} [(\Delta\theta_i^u)^2 + (\Delta\theta_i^v)^2]$$

- … with an associated uncertainty, i.e. standard error:

$$\sigma_{\tilde{A}}^2 = \frac{1}{2N}\left(\mu_4 - \frac{2N-3}{2N-1}A^2\right) \approx \frac{1}{2N}\left(\left[3 - \frac{2N-3}{2N-1}\right]A^2\right)$$

- … which translates to g and to δ:

$$\Rightarrow \quad \sigma_g^2 = \sigma_{\tilde{A}}^2 \left(\left.\frac{\mathrm{d}G}{\mathrm{d}A}\right|_{A=\tilde{A}_{\mathrm{out}}} \right)^2 = \frac{\sigma_{\tilde{A}}^2}{\tau_w^2(\tilde{A}_{\mathrm{out}})} \quad \Rightarrow \quad \sigma_\delta^2 = \sigma_g^2 \frac{\pi^2}{6a^2P} = \frac{1}{2N}\left(\tilde{\mu}_4 - \frac{2N-3}{2N-1}\tilde{A}_{\mathrm{out}}^2\right)\frac{1}{\tau_w^2(\tilde{A}_{\mathrm{out}})}\frac{\pi^2}{6a^2P}$$

Statistical limitations of scattering proton CT

- Relating number of protons N to dose at water center, D_{center}, we get for the uncertainty of δ

$$\sigma_\delta = \left[\frac{\tilde{\mu}_4 - \tilde{A}_{\text{out}}^2}{2\tau_w^2(\tilde{A}_{\text{out}})}\right]^{1/2} \left[\frac{\pi^2\left(S(E_{\text{centre}}) + \kappa\gamma E_{\text{centre}}\right)}{6a^4 \exp(-\kappa r)\,\rho D_{\text{centre}}}\right]^{1/2}$$

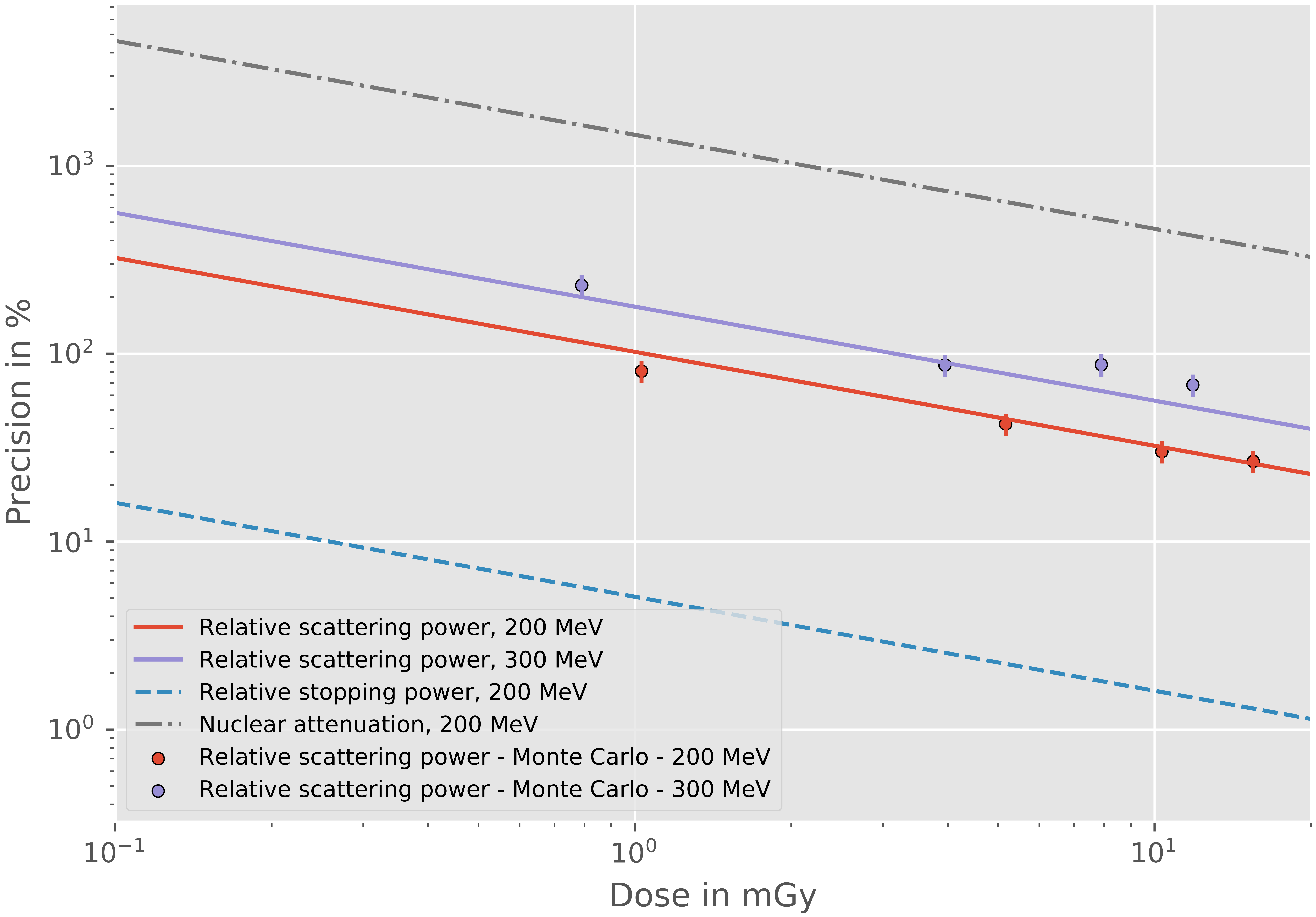
- For comparison, intrinsic noise in energy-loss proton CT[1]

$$\sigma_{\eta_e} = \frac{\sigma_{\text{E}_{\text{out}}}}{S(E_{\text{out}})} \left[\frac{\pi^2\left(S(E_{\text{centre}}) + \kappa\gamma E_{\text{centre}}\right)}{6a^4 \exp(-\kappa r)\,\rho D_{\text{centre}}}\right]^{1/2}$$

where $\sigma_{\text{E}_{\text{out}}}$ is due to energy straggling.

[1] Schulte, R. W., Bashkirov, V., Klock, M. C. L., Li, T., Wroe, A. J., Evseev, I., … Satogata, T. (2005). Density resolution of proton computed tomography. Medical Physics, 32(4), 1035–1046.

Statistical limitations of scattering proton CT

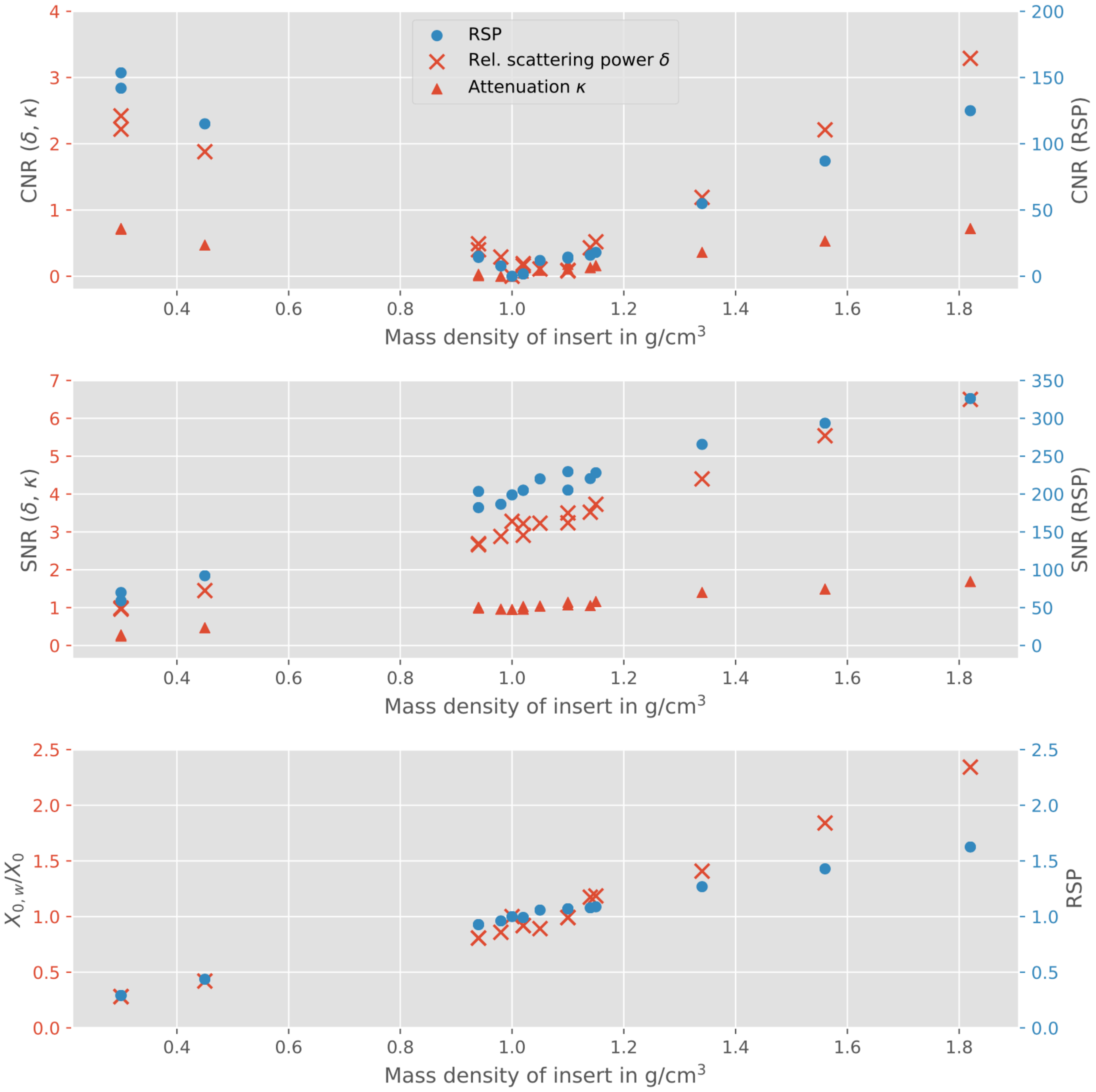


Data points are root mean square errors from 110 independent Monte Carlo simulations

Scattering proton CT is about one order of magnitude noisier than energy-loss proton CT

… mainly because estimating sample variance is more difficult than sample mean.

Gammex phantom: image quality



Contrast-to-noise

and signal-to-noise

are inferior compared to energy-loss proton CT

… although intrinsic contrast in scattering proton CT is higher.

Conclusion

- Filtered back projection reconstruction feasible when using **relative** scattering power
- Most likely path can be incorporated e.g. via distance driven binning.
- Dependence of relative scattering power on angular variance leads to cupping artifacts in dense objects.
- Noise much higher than in energy-loss proton CT
- Imaging system simpler because it needs no energy detector.

Acknowledgements

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Thank you