A Comparison of String Averaging and Component Averaged Row Projection Algorithms Used in Image Reconstruction

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Background

- Problem statement: Compare experimental convergence rates of string averaging projections (SAP) and component averaged row projections (CARP), when relaxation parameter, string length, and sparsity are varied, with the goal of optimizing the relaxation parameter choice.

- Originally an undergraduate extracurricular project, funded by the NIU Office of Student Engagement and Experiential Learning.

- It is now being worked on as a transition toward PhD programs in applied mathematics or a related field.

Introduction

- When reconstructing an image, a linear system of the form $Ax = b$ is used, where A_{ij} represents the length of track of proton i through voxel j, b represents the water equivalent path length vector, and x represents the relative stopping power (RSP) vector.

- Since each track only passes through relatively few voxels, the matrix A is very sparse.

- Iterative algorithms, mainly variations of the algebraic reconstruction technique (ART), are used to solve for x from known A and b.

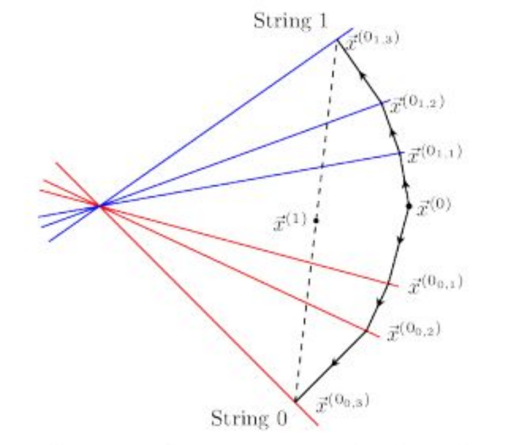
Methods

- Use small, noiseless system of equations (10000 x 300 matrix for A), with a known solution; the ratio of voxels to tracks is approximately equal to that used in the much larger systems used in pCT reconstruction.

- A known solution, x, is generated randomly with all components between 0 and 2, to be consistent with actual RSP values.

- Similarly, components of A are between 0 and 2, consistent with approximate path lengths (in mm) in 1 mm^3 voxels. Every row has the same sparsity.

Algorithms: SAP and CARP

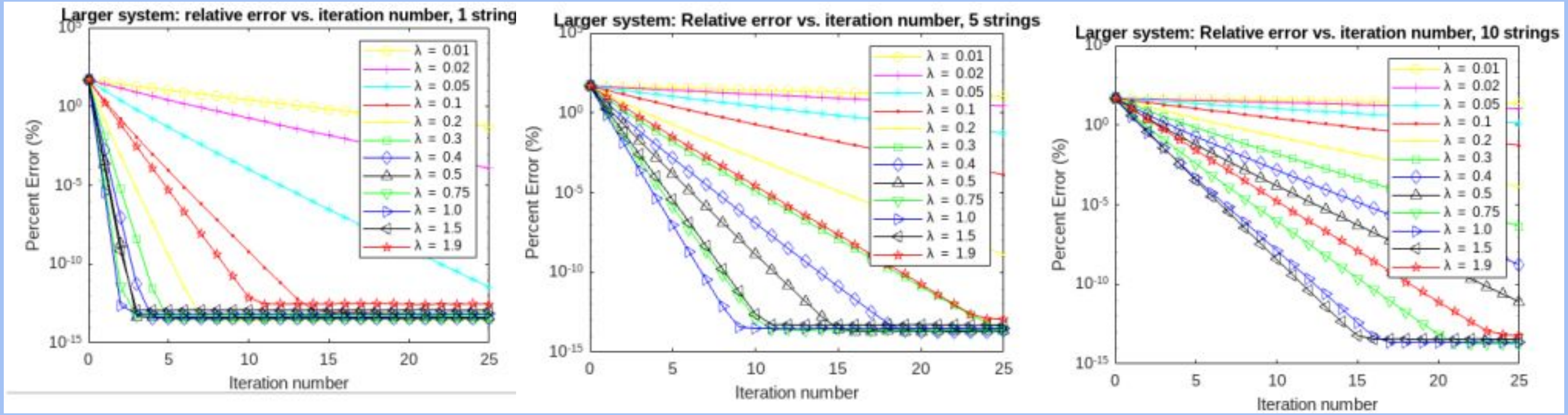


- When applying the iterative algorithms, we let x^0, the starting value of x, to be (1, 1, …, 1).

- For our experiments with SAP, we used equal weights for each string, and therefore equal weights for each component, as well.

- CARP weights each component based on information about the sparsity of each column of A corresponding to the components.

Results: SAP with 90% sparsity



- The percent error is defined by

$$100 \cdot \frac{\sum_j |x_j - x_j^k|}{\sum_j x_j}$$

where x^k is the current iterate and x is the true solution (formula adopted by Penfold and Censor in 2015 pCT paper).

Results: SAP with 90% sparsity (continued)

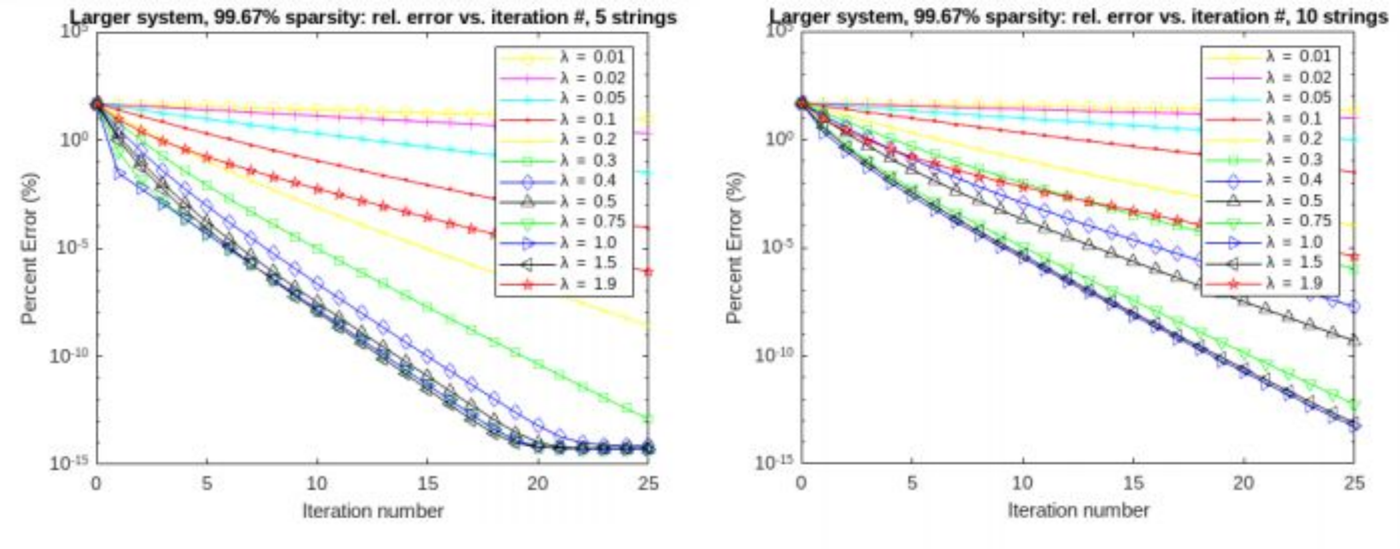
- For some values of the relaxation parameter lambda, rapid initial convergence is observed for all choices of numbers of strings.

- The preferred values of the relaxation parameter (for the values we tested) tend to be between between 0.3 and 1.5, with more variation in convergence rates when there were more strings.

- Because we typically reach a clinically reasonable error after relatively few iterations, there is more work to be done to determine how the relaxation parameter and string length should be chosen.

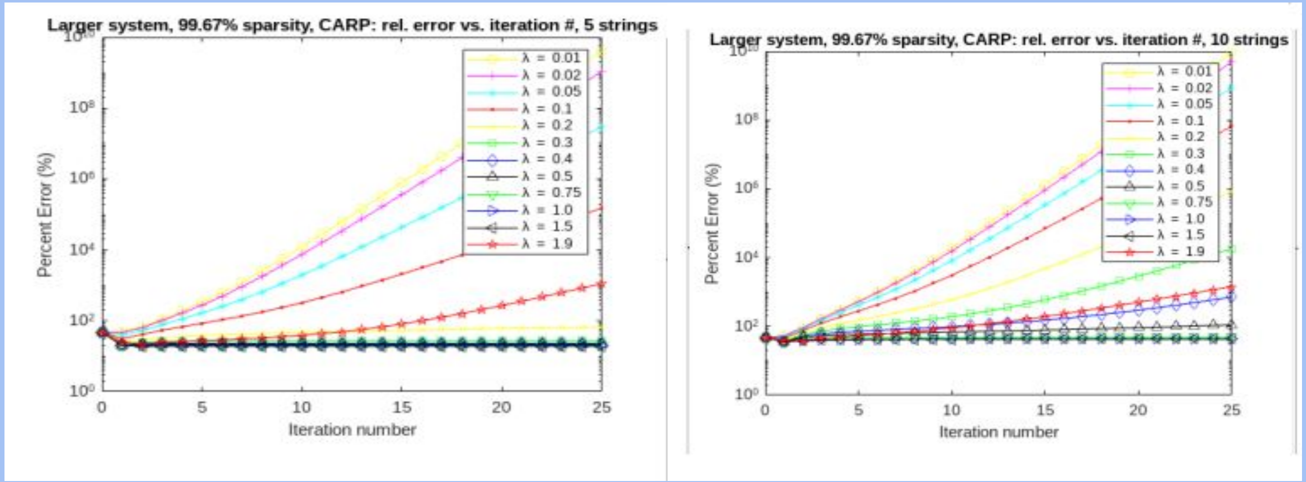
Results: SAP with 99.67% sparsity

- Here, each track passes through exactly 1 voxel.



- Convergence takes longer, but adequate accuracy is achieved in about the same amount of time.

Preliminary results: CARP, 99.67% sparsity



- For many values of lambda, the error diverges; for values near 1.0, there is convergence, but the error is still so large as to eliminate any chance of practical use.

- A good way to rectify this is likely to use larger systems of equations, which will also allow for the greater sparsity used in practice.

Additional Results: SAP and CARP

- In situations with low sparsity or few strings, CARP performs equivalently to SAP, since there will be a nonzero component of A in each column in every string.

- Because CARP frequently performs equivalently to SAP in small systems, it was difficult to detect the computational advantages of CARP, especially with the small system used here.

- If every equation is placed into a single string, then we get the well-known algorithm ART from SAP.

Future work

- Use larger, sparser systems, more consistent with practical use, for SAP and CARP.
- Compare to CARP-Conjugate Gradient (CARP-CG), and systems with noise.
- Vary relaxation parameter from iteration to iteration, or consider values outside the interval (0, 2) that is typically used.
- The end goal is to select the optimal relaxation parameter for fast convergence and low runtime, and develop an understanding of what parameters are most likely to impact the optimal relaxation parameter for each algorithm used.

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Thank you!

Questions?