Northern Illinois University

pCT Image Quality Comparisons with CARP, DROP, and TVS

Christina Sarosiek

7th Annual Loma Linda Workshop

August 3, 2021

Iterative solvers of large sets of linear equations are currently the starting point for PCT image reconstruction

- Equations of the form , A(RSP) =WEPL are used to solve for the RSP vector where A = known matrix of order 10^6 x 10^8
- CARP and DROP are the most common algorithms used for PCT image reconstruction and requires a known (or guessed) relaxation parameter to obtain solutions.

CARP

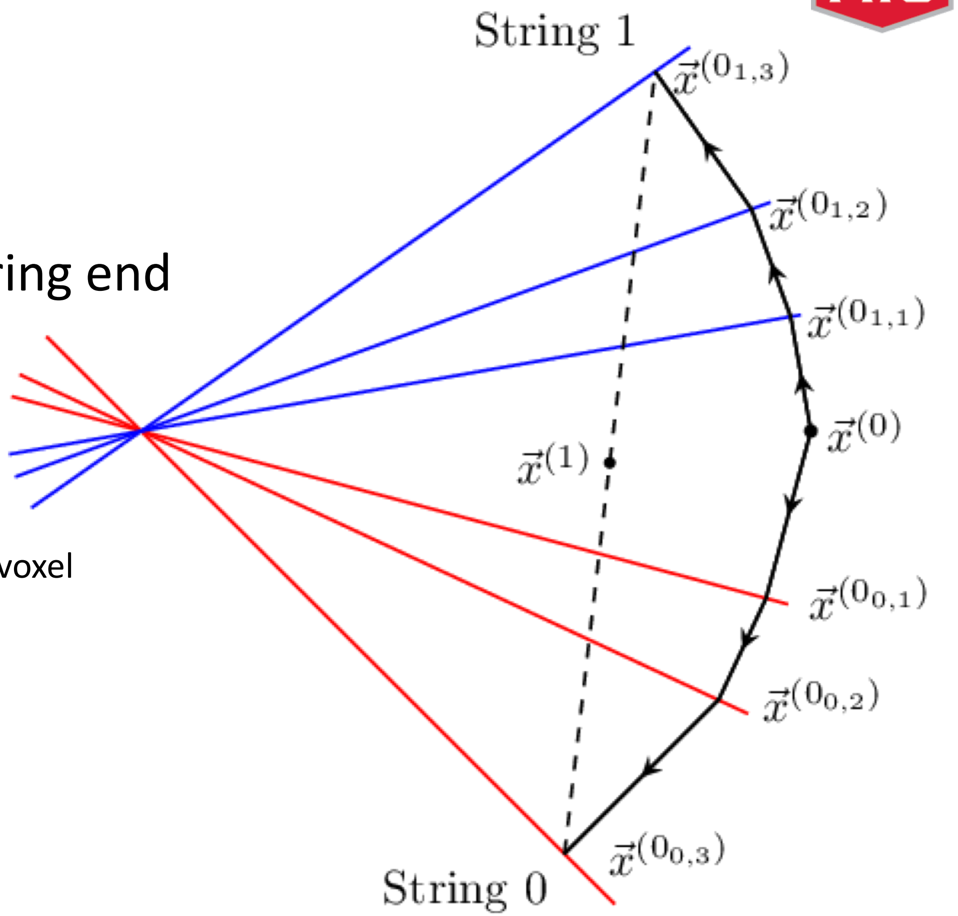
- CARP is a string averaging method
- Split the hyperplanes into subsets, called strings
- Projection sequentially on to each hyperplane in a string
- All strings can be done in parallel
- The subsequent iteration is the convex combination of the string end points.

$$\vec{x}^{(k_{s,p+1})} = \vec{x}^{(k_{s,p})} + \lambda \sum_i \frac{b_i - \langle \vec{a}_i, \vec{x}^{(k_{s,p})} \rangle}{\|\vec{a}_i\|} \vec{a}_i^T$$

$$\vec{x}^{(k+1)} = \vec{x}^{(k)} + \boldsymbol{C} \sum_{s=0}^{S-1} \vec{x}^{(k_{s,P})}$$

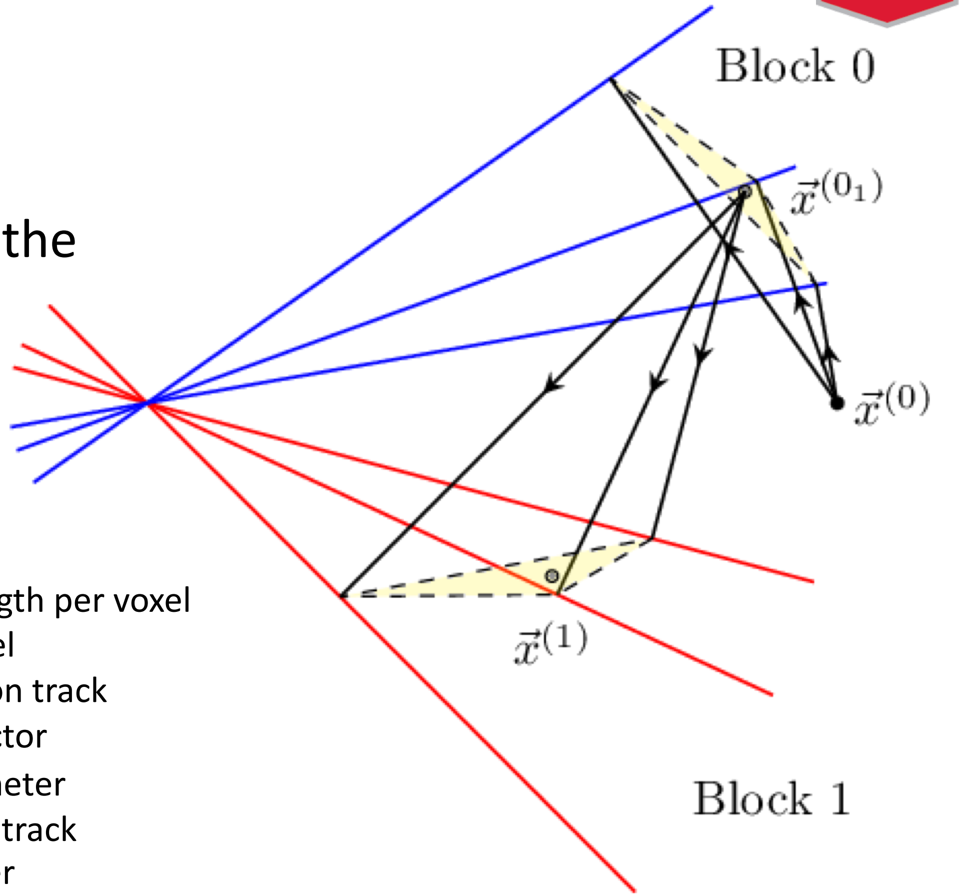
$$\boldsymbol{C} = \operatorname*{diag}_{1 \leq j \leq n} \left(\frac{1}{c_j} \right)$$

a_i= proton path length per voxel
$\vec{x}$ = RSP in each voxel
b_i = WEPL per proton track
$\boldsymbol{C}, c_j$= weighting factor
λ= relaxation parameter
i = index for proton track
k = iteration number
s = string number
p = projection number



DROP

- DROP is a block iterative method
- Split the hyperplanes into subsets, called blocks
- Project simultaneously on to each hyperplane in a block
- Then use the convex combination of each projection point as the initial solution for the next block.
- The subsequent iteration is the solution of the final block



$$\vec{x}^{(k_{p+1})} = \vec{x}^{(k_p)} + \lambda \sum_{i=1}^{S_p} \frac{b_i - \langle \vec{a}_i, \vec{x}^{(k_p)} \rangle}{\|\vec{a}_i\|^2} \vec{a}_i^T$$

$$\boldsymbol{D} = \underset{1 \le j \le n}{\operatorname{diag}} \left(\min \left(\frac{1}{d_j} \right) \right)$$

a_i= proton path length per voxel
$\vec{x}$ = RSP in each voxel
b_i = WEPL per proton track
$\mathbf{D}$, d_j= weighting factor
λ= relaxation parameter
i = index for proton track
k = iteration number
p = partial iteration number

Questions we wish to address

- What are the differences in image quality when using CARP vs. DROP, with and without TVS?
- What does TVS provide in terms of image quality?
- Are we currently using the best relaxation parameter for our system of equations and number of strings/blocks?
- How many proton tracks per voxel are needed for adequate image quality?

The UCSC/LLU Proton CT Scanner

- Allows direct (most accurate) measurement of RSP of each voxel
- This allows for reduction of beam-specific range margins during treatment planning
- This will lead to a reduction of NTCP on structures close to the CTV (see presentation by Andrew Best)

- Single particle tracking
- 1 MHz protons
- 9 x 36 cm^2 field of view
- 1 RPM continuous rotation
- Nominal run length of 6 min
- 360 million protons
- About 120 million protons passing through about $3x10^6$ voxels are recorded after preprocessing cuts.

Image Reconstruction Algorithms under Study

- CARP and DROP (as mentioned earlier)
- Total variation superiorization (TVS) to smooth the image
  - Algorithm allows for smoothing without reducing spatial resolution.
  - CARP+TVSi: TVS is performed after each CARP iteration
  - DROP+TVSi: TVS is performed after each DROP iteration
  - DROP+TVSb: TVS is performed after each DROP Block
- Computed on a large cluster (called Gaea) with several hundred CPUs and 120 GPUs.
- 40 Strings used for CARP
- 40 Strings used for DROP
- 20 iterations per image (or RSP solution)
- Total execution time for 120 M proton histories (10 iterations) = 1 minute (after preprocessing)

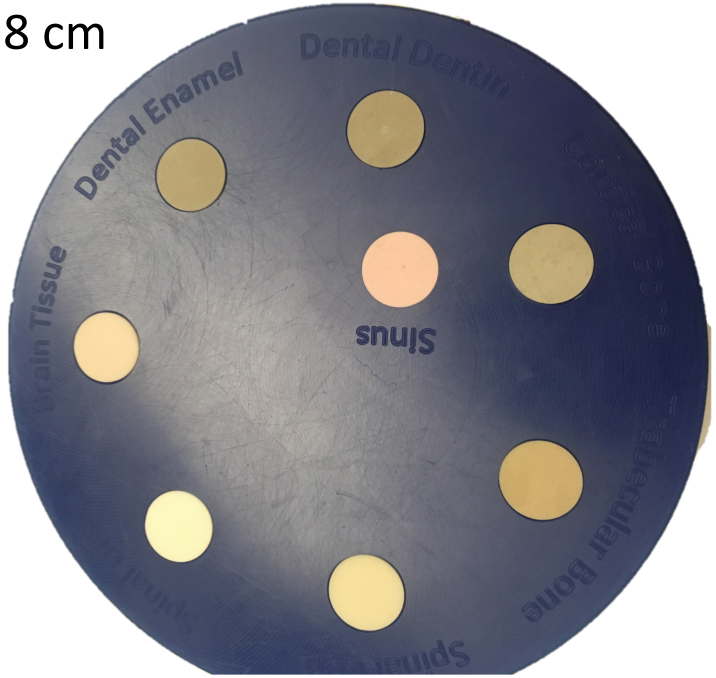
Image Quality

- Three image quality metrics: RSP Accuracy, Spatial Resolution, Noise (CNR, SNR)
- These metrics have direct correlations to the clinical uses of the images.
- Treatment planning:
  - High spatial resolution and low noise are necessary for contouring tumor volumes and organs at risk
  - High RSP accuracy is necessary for accurate proton range predictions
- Image quality metrics calculated on two phantoms; line pair phantom and RSP phantom
- Developed a cost function to combine the 3 metrics into a single quality metric.

Phantoms

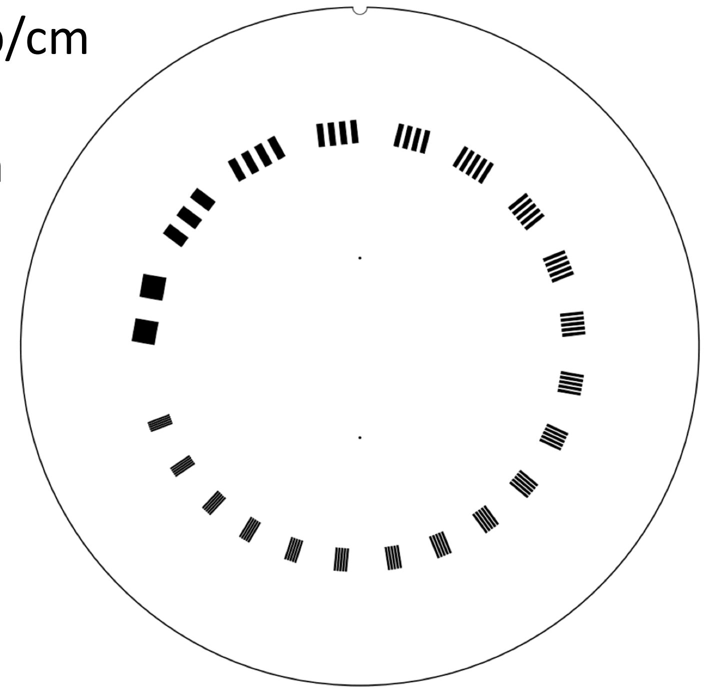
Peg (George) phantom:

- Blue bolus wax background
- Eight tissue-equivalent inserts
- Diameter: 15 cm
- Height: 4 cm
- Insert diameter: 1.8 cm



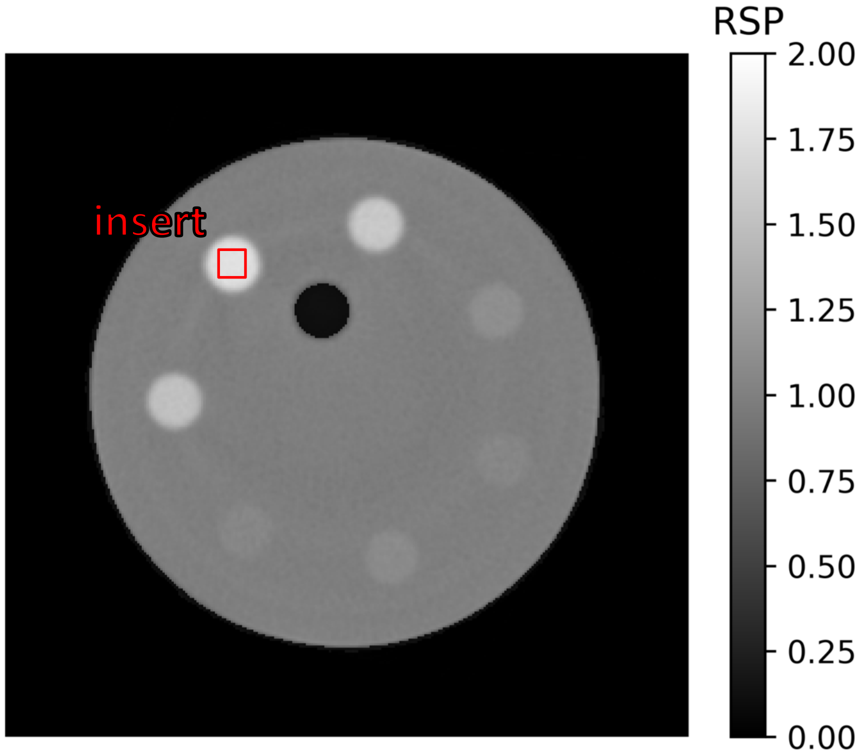
CATPHAN Line Pair phantom:

- Acrylic background
- 21 line pairs
- Diameter: 12 cm
- Height: 4 cm
- Insert width: 1 to 21 lp/cm
- Insert length: 1 cm
- Insert thickness: 4 mm



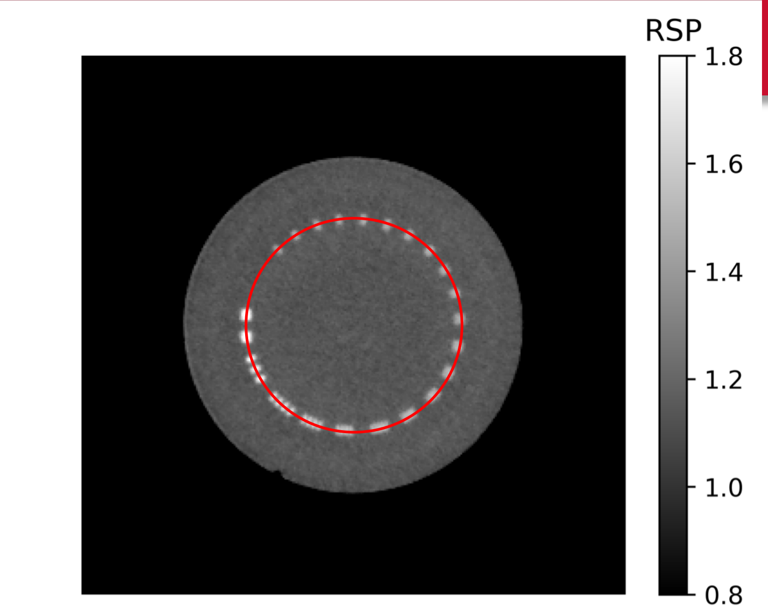
RSP Accuracy

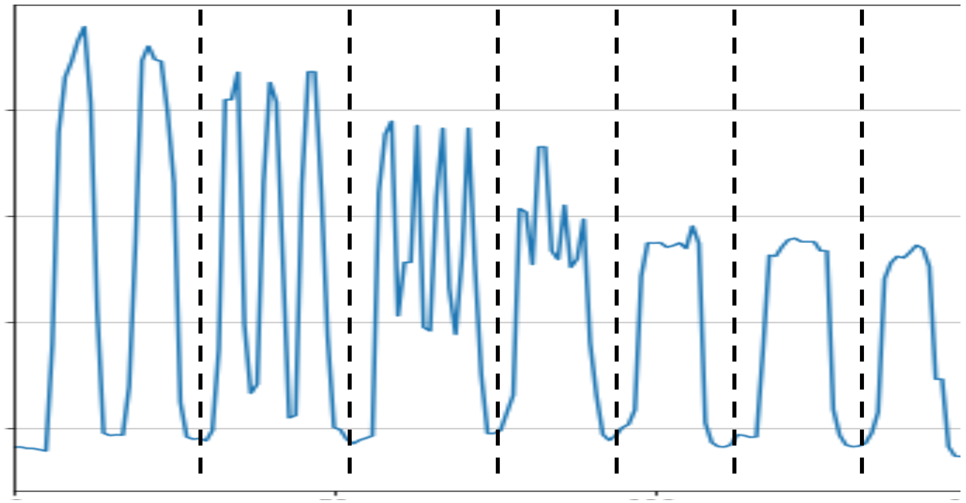
- Take a region of interest (ROI) in each insert and background
- Calculate the mean RSP in the ROI
- Target quantity: Percent error <±1%
  - calculated by $\left|\frac{RSP_{ROI}-RSP_{truth}}{RSP_{truth}}\times 100\right|$
- Mean absolute percent error (MAPE) is calculated by taking the average of the percent errors from all inserts.
- To ensure the error is systematic for all inserts, we also calculate the standard deviation.



Spatial Resolution

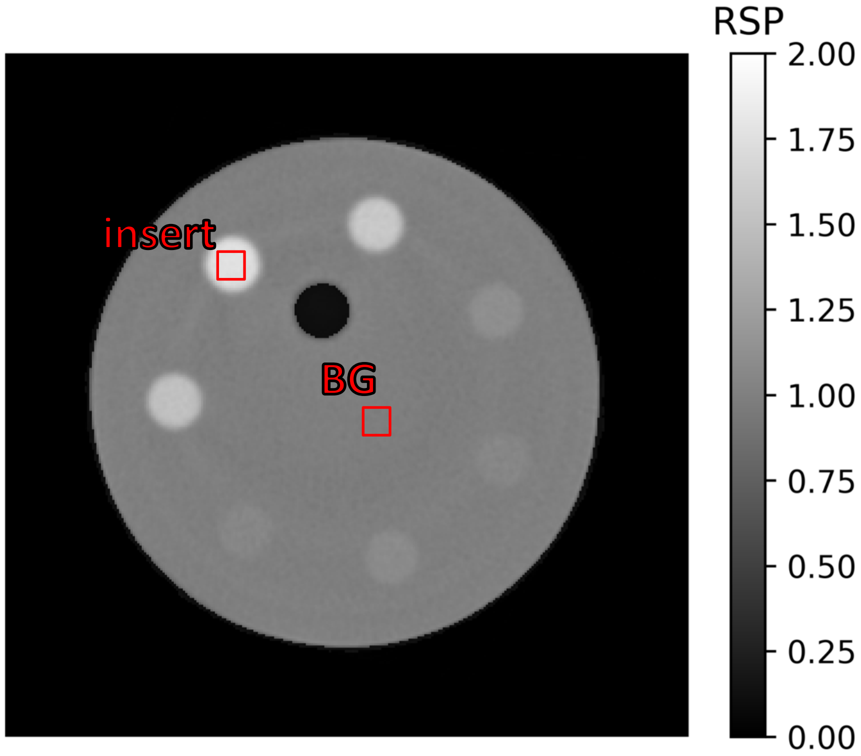
- Modulated transfer function (MTF)
- Circular profile through the inserts
- Locate the peaks and valleys in each of the pairs
- For each line pair, calculate the MTF:
  - $\mathrm{MTF}_{\mathrm{LP}} = \dfrac{[\mathrm{RSP}_{\mathrm{peak}} - \mathrm{RSP}_{\mathrm{valley}}]_{\mathrm{LP}}}{[\mathrm{RSP}_{\mathrm{peak}} - \mathrm{RSP}_{\mathrm{valley}}]_{\mathrm{LP=1\,lp/cm}}}$
  - The denominator normalizes the MTF of large spacings to one
- Plot the $\mathrm{MTF}_{\mathrm{LP}}$ vs LP
- As the spacing between the line pairs gets smaller, partial volume effects take over and the difference between the RSP in the peak and valley gets smaller.
- The spatial resolution is defined as the MTF-10%, the line pair where the MTF reaches 10% or 0.1
- Target quantity: 1 mm or 5 lp/cm





Noise

- Two noise metrics: Signal-to-noise ratio (SNR), Contrast-to-noise (CNR)
- These metrics measure how well you can detect low contrast objects in the presence of noise
- $\mathrm{SNR} = \frac{\overline{\mathrm{RSP}}}{\sigma}$
- $\mathrm{CNR} = \frac{|\overline{\mathrm{RSP}}_{\mathrm{insert}} - \overline{\mathrm{RSP}}_{\mathrm{BG}}|}{\sqrt{\sigma^2_{\mathrm{insert}} + \sigma^2_{\mathrm{BG}}}}$
- Target noise: 1% of the signal, SNR = 100, CNR dependent on numerator



Global Cost Function

- These metrics can work in opposition of each other. E.g. low noise -> poor spatial resolution
- We developed a weighted cost function to combine the three metrics

If $f_i > t_i$, then $c_i = f_i$
Else if $f_i < t_i$, then $c_i = {f_i}^2$

$$C = w_{acc}c_{acc} + w_{SR}c_{SR} + w_{noise}c_{noise}$$

$$w_{acc} = 40, w_{SR} = 30, w_{noise} = 30$$

$f_{acc} = MAPE + \sigma_{MAPE}$	$t_{acc} = 0.015$
$f_{SR} = \frac{1}{\mathrm{MTF}-10\%}$	$t_{SR} = 0.02$
$f_{noise} = \frac{1}{SNR} = \frac{\sigma}{\overline{RSP}}$	$t_{noise} = 0.01$

- By squaring f for f < t, we are allowing images that meet the target value to have a lessor penalty.
- The target values can be adjusted for specific use case
- The weights were chosen for a general use case, but could be adjusted for a specific use case.

General Comments

- Noise and Spatial resolution are in direct competition with each other. Optimizing Lambda for best spatial resolution yields higher noise. (e.g., lower SNR and CNR)
- After about six iterations, the RSP accuracy for all images is most dependent on the relaxation parameter.
- Inappropriate choice of the relaxation parameter can yield a larger spread in RSP accuracy for the eight inserts which is penalized through the cost function.
- When judging accuracy, it is therefore important to account for both the MAPE and the accuracy of each individual material reflected in σ_{MAPE}.

Results

- Overall, we can achieve good image quality with any algorithm or dose level assuming the correct relaxation parameter is selected. (MTF-10% is given in line pairs/cm)

Run Length	Algorithm	λ	IN	MAPE	Max % Err	Min % Err	MTF-10%	SNR	CNR	Cost
6 min	CARP	0.002	7	0.648	1.41 (Disc)	0.028 (BG)	3	98.57	4.56	0.0131
	CARP+TVSi	0.08	1	0.525	1.03 (Disc)	0.03 (Dentin)	4	42.98	1.96	0.0145
	DROP	0.14	5	0.452	1.26 (Disc)	0.006 (BG)	3	101.6	4.50	0.0101
	DROP+TVSi	0.05	15	0.528	1.17 (Disc)	0.10 (BG)	3	173.2	7.84	0.0100
	DROP+TVSb	0.16	5	0.307	0.64 (TB)	0.005 (Sinus)	2	1737.6	72.2	0.015
3 min	CARP	0.01	4	0.933	1.84 (Sinus)	0.204 (CB)	2	62.45	3.26	0.0167
	CARP+TVSi	0.04	16	1.16	5.02 (Sinus)	0.073 (Dentin)	5	211.1	12.6	0.0108
	DROP	0.24	3	0.629	1.36 (Disc)	0.067 (CB)	2	65.11	3.42	0.0169
	DROP+TVSi	0.05	17	0.883	2.00 (Sinus)	0.172 (CB)	2	93.88	5.02	0.0153
	DROP+TVSb	0.9	14	1.13	3.97 (Sinus)	0.138 (Cord)	5	160.0	9.21	0.0093
12 min	CARP	0.005	2	0.661	1.84 (Sinus)	0.018 (Enamel)	3	144.4	6.97	0.0101
	CARP+TVSi	0.01	1	0.519	1.18 (Sinus)	0.004 (Dentin)	3	517.0	28.1	0.0100
	DROP	0.60	1	0.337	0.668 (Disc)	0.106 (Dentin)	3	155.2	7.34	0.0100
	DROP+TVSi	0.05	16	0.426	0.863 (Disc)	0.069 (Enamel)	3	351.9	17.9	0.0100

Results

- For CARP, TVS decreased the SNR and CNR by over a factor of 2 but with somewhat better spatial resolution and better Mean Average Percent Error.

Run Length	Algorithm	λ	IN	MAPE	Max % Err	Min % Err	MTF-10%	SNR	CNR	Cost
6 min	CARP	0.002	7	0.648	1.41 (Disc)	0.028 (BG)	3	98.57	4.56	0.0131
	CARP+TVSi	0.08	1	0.525	1.03 (Disc)	0.03 (Dentin)	4	42.98	1.96	0.0145
	DROP	0.14	5	0.452	1.26 (Disc)	0.006 (BG)	3	101.6	4.50	0.0101
	DROP+TVSi	0.05	15	0.528	1.17 (Disc)	0.10 (BG)	3	173.2	7.84	0.0100
	DROP+TVSb	0.16	5	0.307	0.64 (TB)	0.005 (Sinus)	2	1737.6	72.2	0.015
3 min	CARP	0.01	4	0.933	1.84 (Sinus)	0.204 (CB)	2	62.45	3.26	0.0167
	CARP+TVSi	0.04	16	1.16	5.02 (Sinus)	0.073 (Dentin)	5	211.1	12.6	0.0108
	DROP	0.24	3	0.629	1.36 (Disc)	0.067 (CB)	2	65.11	3.42	0.0169
	DROP+TVSi	0.05	17	0.883	2.00 (Sinus)	0.172 (CB)	2	93.88	5.02	0.0153
	DROP+TVSb	0.9	14	1.13	3.97 (Sinus)	0.138 (Cord)	5	160.0	9.21	0.0093
12 min	CARP	0.005	2	0.661	1.84 (Sinus)	0.018 (Enamel)	3	144.4	6.97	0.0101
	CARP+TVSi	0.01	1	0.519	1.18 (Sinus)	0.004 (Dentin)	3	517.0	28.1	0.0100
	DROP	0.60	1	0.337	0.668 (Disc)	0.106 (Dentin)	3	155.2	7.34	0.0100
	DROP+TVSi	0.05	16	0.426	0.863 (Disc)	0.069 (Enamel)	3	351.9	17.9	0.0100

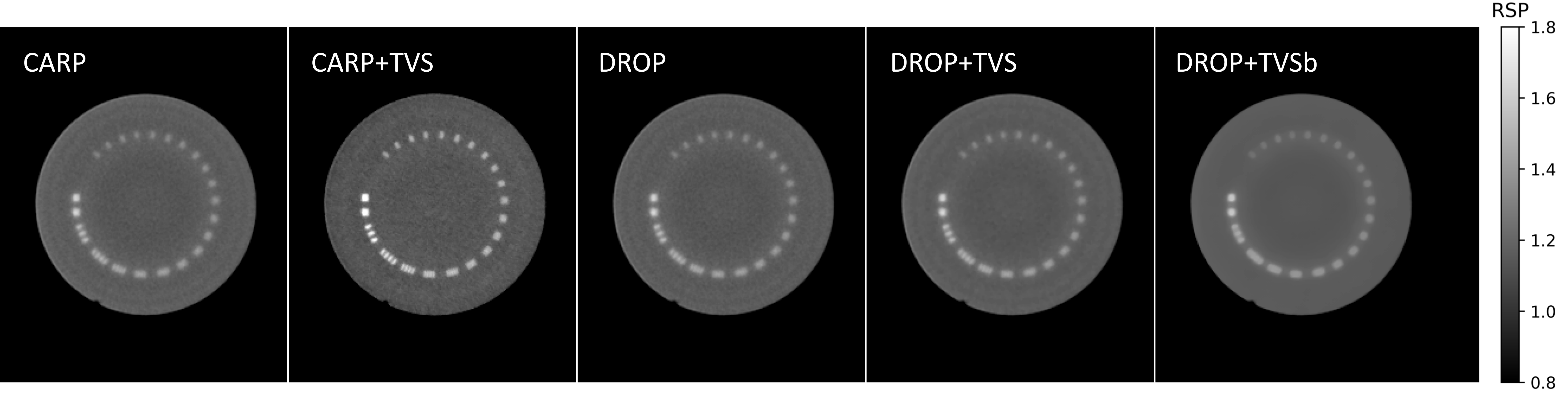
Results

- Using DROP, adding TVS improved the SNR and CNR by 70% with no decrease in spatial resolution, but the Mean Average Percent Error (average of the 8 absolute errors) was slightly higher with TVS.

Run Length	Algorithm	λ	IN	MAPE	Max % Err	Min % Err	MTF-10%	SNR	CNR	Cost
6 min	CARP	0.002	7	0.648	1.41 (Disc)	0.028 (BG)	3	98.57	4.56	0.0131
	CARP+TVSi	0.08	1	0.525	1.03 (Disc)	0.03 (Dentin)	4	42.98	1.96	0.0145
	DROP	0.14	5	0.452	1.26 (Disc)	0.006 (BG)	3	101.6	4.50	0.0101
	DROP+TVSi	0.05	15	0.528	1.17 (Disc)	0.10 (BG)	3	173.2	7.84	0.0100
	DROP+TVSb	0.16	5	0.307	0.64 (TB)	0.005 (Sinus)	2	1737.6	72.2	0.015
3 min	CARP	0.01	4	0.933	1.84 (Sinus)	0.204 (CB)	2	62.45	3.26	0.0167
	CARP+TVSi	0.04	16	1.16	5.02 (Sinus)	0.073 (Dentin)	5	211.1	12.6	0.0108
	DROP	0.24	3	0.629	1.36 (Disc)	0.067 (CB)	2	65.11	3.42	0.0169
	DROP+TVSi	0.05	17	0.883	2.00 (Sinus)	0.172 (CB)	2	93.88	5.02	0.0153
	DROP+TVSb	0.9	14	1.13	3.97 (Sinus)	0.138 (Cord)	5	160.0	9.21	0.0093
12 min	CARP	0.005	2	0.661	1.84 (Sinus)	0.018 (Enamel)	3	144.4	6.97	0.0101
	CARP+TVSi	0.01	1	0.519	1.18 (Sinus)	0.004 (Dentin)	3	517.0	28.1	0.0100
	DROP	0.60	1	0.337	0.668 (Disc)	0.106 (Dentin)	3	155.2	7.34	0.0100
	DROP+TVSi	0.05	16	0.426	0.863 (Disc)	0.069 (Enamel)	3	351.9	17.9	0.0100

Image Results of 5 algorithms using “Global” Cost Function

- CARP+TVS visually provides superior spatial resolution.
- Furthermore, the contrast in the line pair phantom, not CNR, is best with CARP+TVS compared to the images from all other algorithms.



Results

- CARP+TVS images show a reduced SNR value with a higher MTF-10% value, suggesting that an SNR target value of 100 is too high and that it would be acceptable to relax the constraint on the noise and instead emphasize higher spatial resolution.

Run Length	Algorithm	λ	IN	MAPE	Max % Err	Min % Err	MTF-10%	SNR	CNR	Cost
6 min	CARP	0.002	7	0.648	1.41 (Disc)	0.028 (BG)	3	98.57	4.56	0.0131
	CARP+TVSi	0.08	1	0.525	1.03 (Disc)	0.03 (Dentin)	4	42.98	1.96	0.0145
	DROP	0.14	5	0.452	1.26 (Disc)	0.006 (BG)	3	101.6	4.50	0.0101
	DROP+TVSi	0.05	15	0.528	1.17 (Disc)	0.10 (BG)	3	173.2	7.84	0.0100
	DROP+TVSb	0.16	5	0.307	0.64 (TB)	0.005 (Sinus)	2	1737.6	72.2	0.015
3 min	CARP	0.01	4	0.933	1.84 (Sinus)	0.204 (CB)	2	62.45	3.26	0.0167
	CARP+TVSi	0.04	16	1.16	5.02 (Sinus)	0.073 (Dentin)	5	211.1	12.6	0.0108
	DROP	0.24	3	0.629	1.36 (Disc)	0.067 (CB)	2	65.11	3.42	0.0169
	DROP+TVSi	0.05	17	0.883	2.00 (Sinus)	0.172 (CB)	2	93.88	5.02	0.0153
	DROP+TVSb	0.9	14	1.13	3.97 (Sinus)	0.138 (Cord)	5	160.0	9.21	0.0093
12 min	CARP	0.005	2	0.661	1.84 (Sinus)	0.018 (Enamel)	3	144.4	6.97	0.0101
	CARP+TVSi	0.01	1	0.519	1.18 (Sinus)	0.004 (Dentin)	3	517.0	28.1	0.0100
	DROP	0.60	1	0.337	0.668 (Disc)	0.106 (Dentin)	3	155.2	7.34	0.0100
	DROP+TVSi	0.05	16	0.426	0.863 (Disc)	0.069 (Enamel)	3	351.9	17.9	0.0100

Results

- DROP +TVSb gave the poorest spatial resolution. However, the CNR is 10 times higher than DROP+TVSi and better RSP accuracy than other algorithms.

Run Length	Algorithm	λ	IN	MAPE	Max % Err	Min % Err	MTF-10%	SNR	CNR	Cost
6 min	CARP	0.002	7	0.648	1.41 (Disc)	0.028 (BG)	3	98.57	4.56	0.0131
	CARP+TVSi	0.08	1	0.525	1.03 (Disc)	0.03 (Dentin)	4	42.98	1.96	0.0145
	DROP	0.14	5	0.452	1.26 (Disc)	0.006 (BG)	3	101.6	4.50	0.0101
	DROP+TVSi	0.05	15	0.528	1.17 (Disc)	0.10 (BG)	3	173.2	7.84	0.0100
	DROP+TVSb	0.16	5	0.307	0.64 (TB)	0.005 (Sinus)	2	1737.6	72.2	0.015
3 min	CARP	0.01	4	0.933	1.84 (Sinus)	0.204 (CB)	2	62.45	3.26	0.0167
	CARP+TVSi	0.04	16	1.16	5.02 (Sinus)	0.073 (Dentin)	5	211.1	12.6	0.0108
	DROP	0.24	3	0.629	1.36 (Disc)	0.067 (CB)	2	65.11	3.42	0.0169
	DROP+TVSi	0.05	17	0.883	2.00 (Sinus)	0.172 (CB)	2	93.88	5.02	0.0153
	DROP+TVSb	0.9	14	1.13	3.97 (Sinus)	0.138 (Cord)	5	160.0	9.21	0.0093
12 min	CARP	0.005	2	0.661	1.84 (Sinus)	0.018 (Enamel)	3	144.4	6.97	0.0101
	CARP+TVSi	0.01	1	0.519	1.18 (Sinus)	0.004 (Dentin)	3	517.0	28.1	0.0100
	DROP	0.60	1	0.337	0.668 (Disc)	0.106 (Dentin)	3	155.2	7.34	0.0100
	DROP+TVSi	0.05	16	0.426	0.863 (Disc)	0.069 (Enamel)	3	351.9	17.9	0.0100

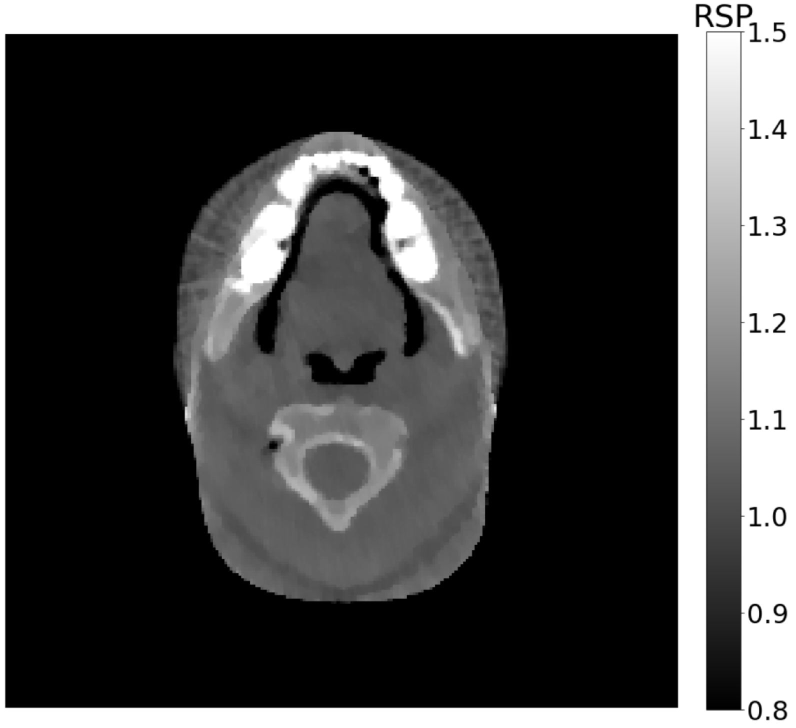
Summary

- Image Quality of proton CT images demonstrate CARP gives higher spatial resolution, DROP gives higher SNR when all 3 metrics are included for "best" λ.

-When an image is optimized for spatial resolution, the SNR and CNR decreases

-Conversely, when an image is optimized for low noise, the spatial resolution worsens

- CARP +TVS shows best overall image quality
- Spatial resolution of DROP+TVS is noticeably worse than CARP+TVS
- RSP accuracy is nearly the same for 3,6, and 12 minute scans (or 0.5, 1.0 and 2.0 mGy), but noise diminishes with higher dose as expected.



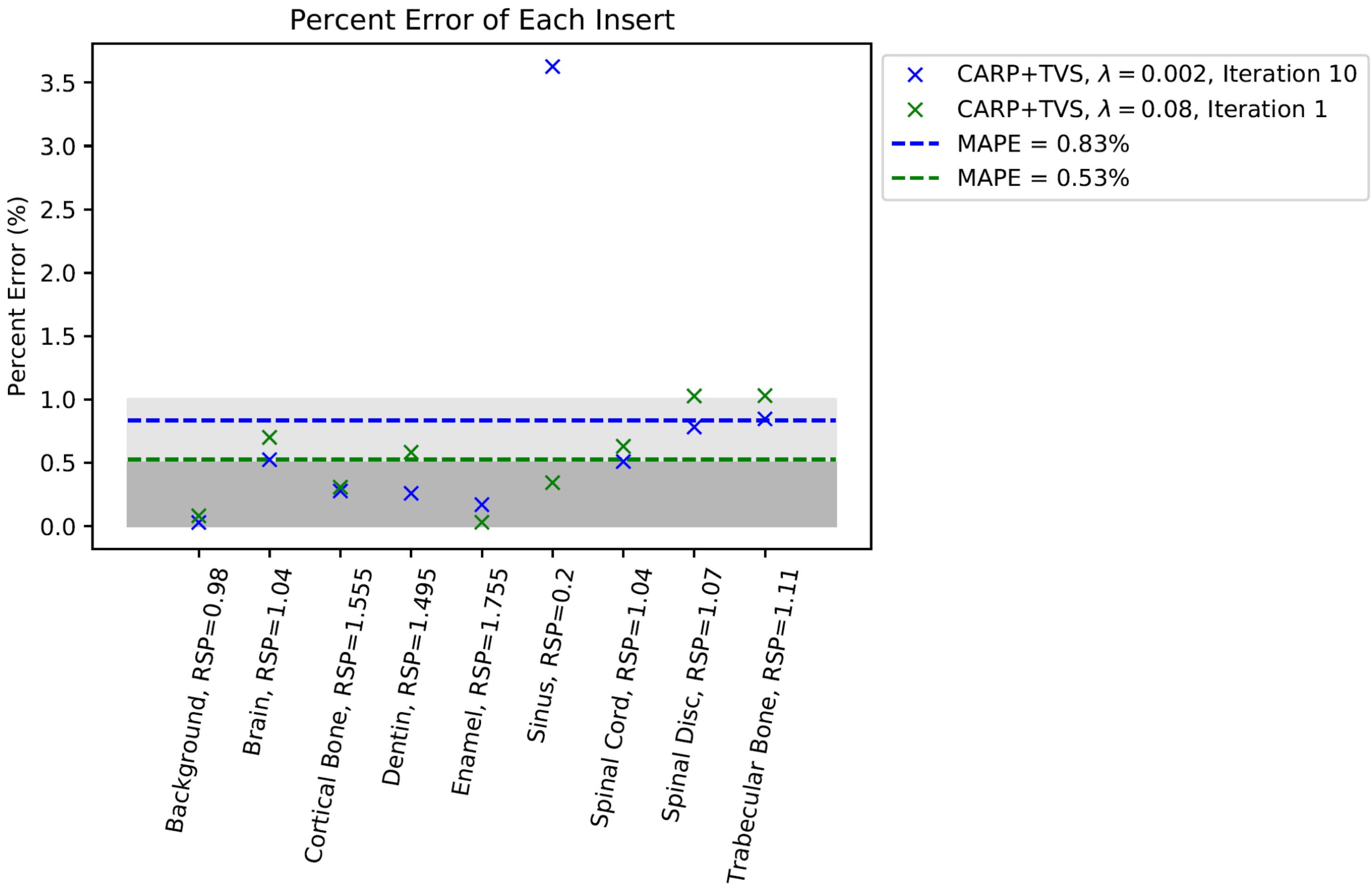
Acknowledgements

- Christina Sarosiek
- Nick Karonis
- Caesar Ordoñez
- Kirk Duffin
- John Winans
- Reinhard Schulte
- Robert Johnson

THANK YOU!

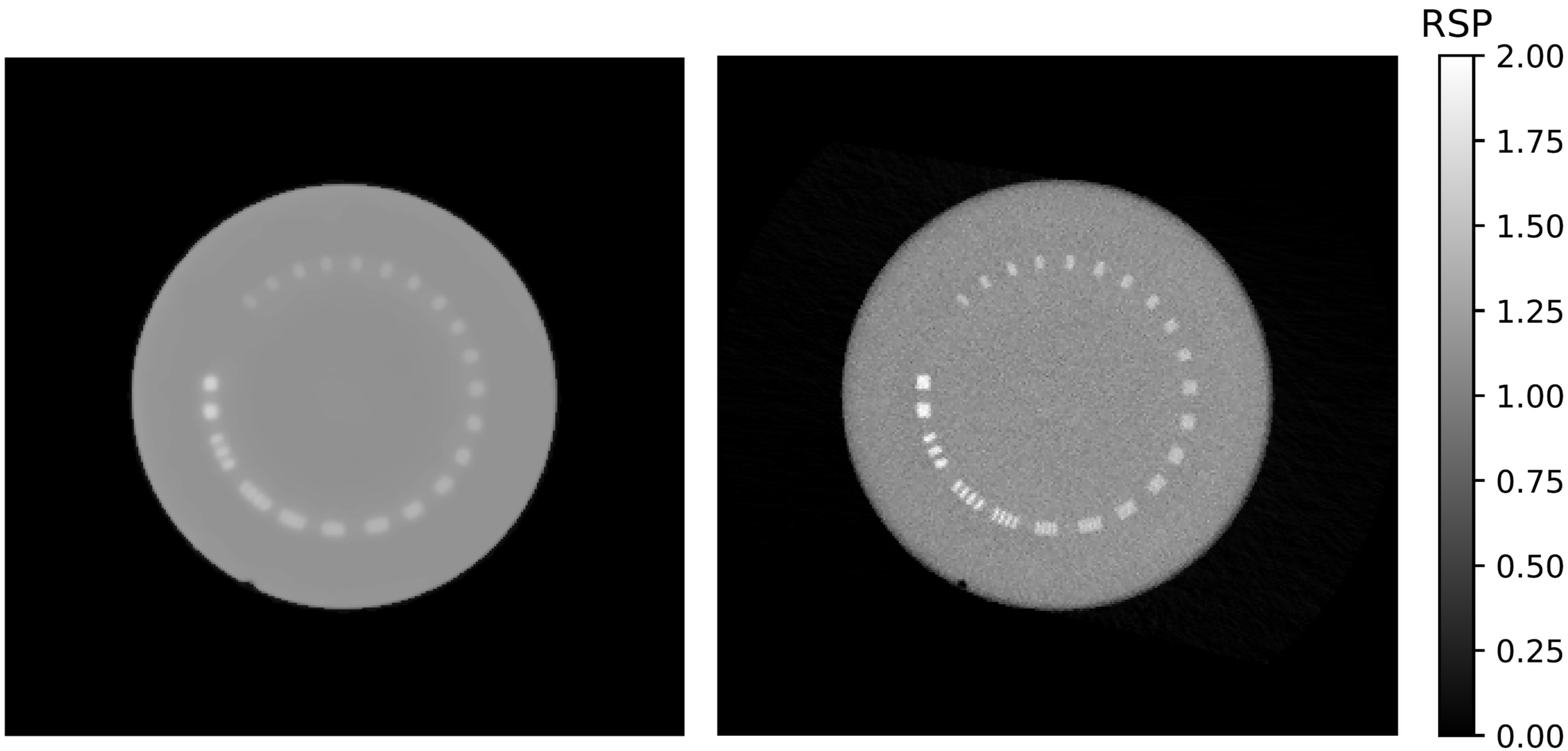
Backup Slides: Accuracy

- It is insufficient to optimize the MAPE, we must also look at the individual densities.
- After ~6 iterations, the accuracy is most dependent on relaxation parameter
- The highest relaxation parameters causes the solution to show rapid variations with iteration and therefore instability in the convergence.



Backup Slides: Spatial Resolution

- Some level of noise is necessary for high spatial resolution



LEFT:

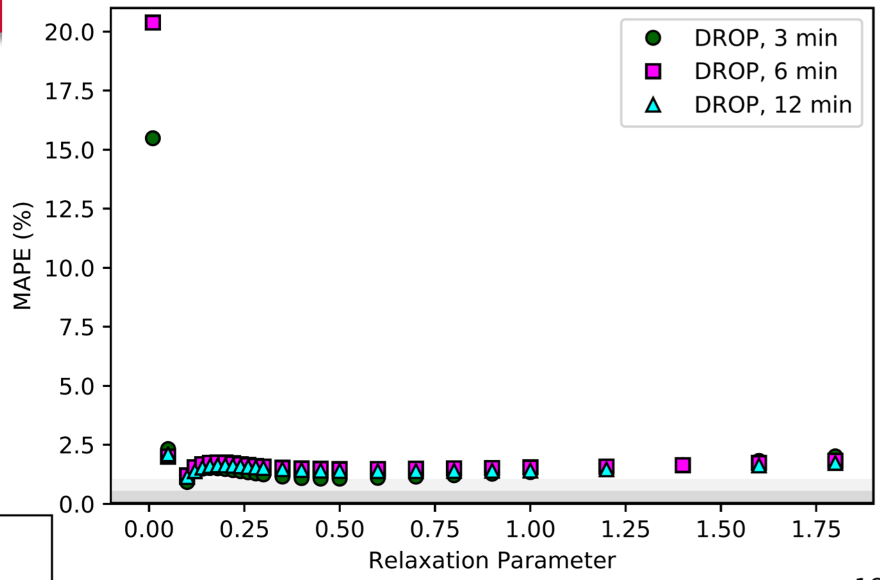
- CARP+TVS, λ = 0.002
- SNR = 4366.7
- MTF-10% = 2 lp/cm

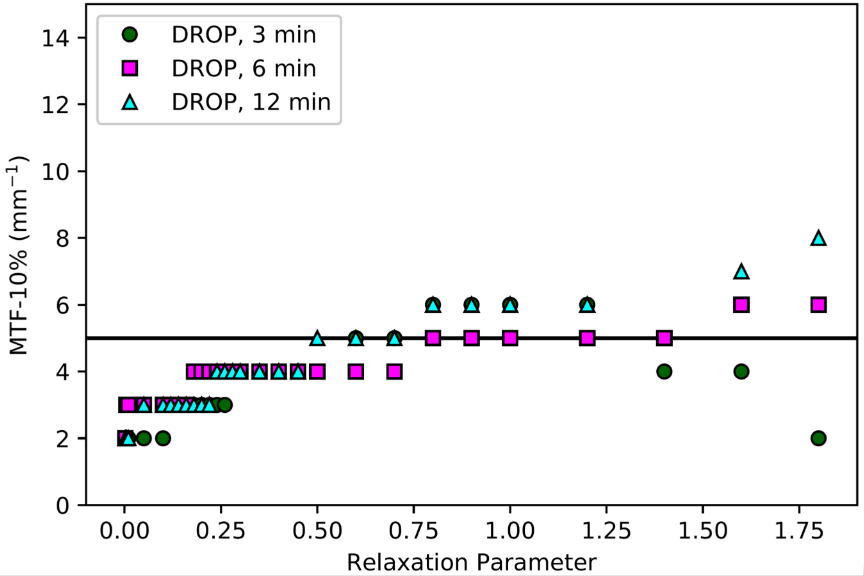
RIGHT:

- CARP+TVS, λ = 0.16
- SNR = 14.19
- MTF-10% = 6 lp/cm

Backup Slides: Dose

- Dose has the largest effect on the noise, especially at low relaxation parameters.





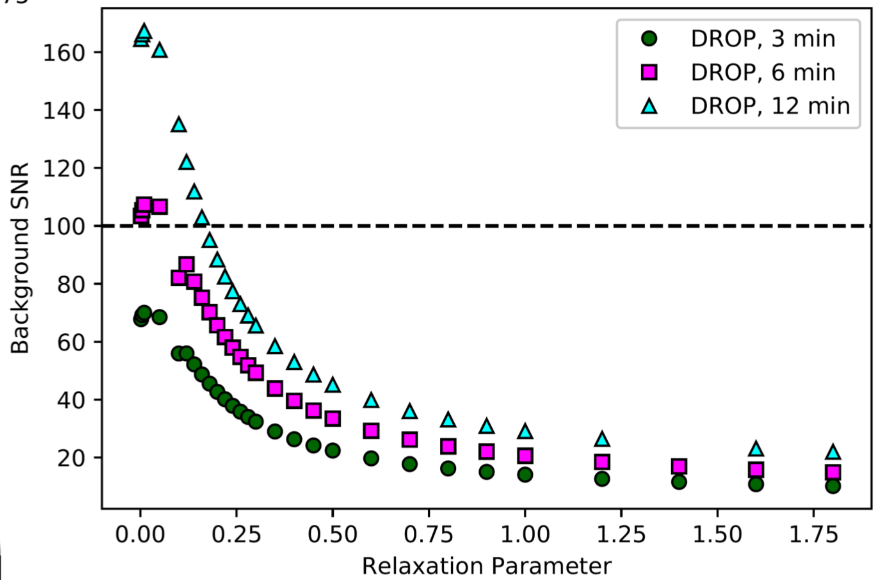
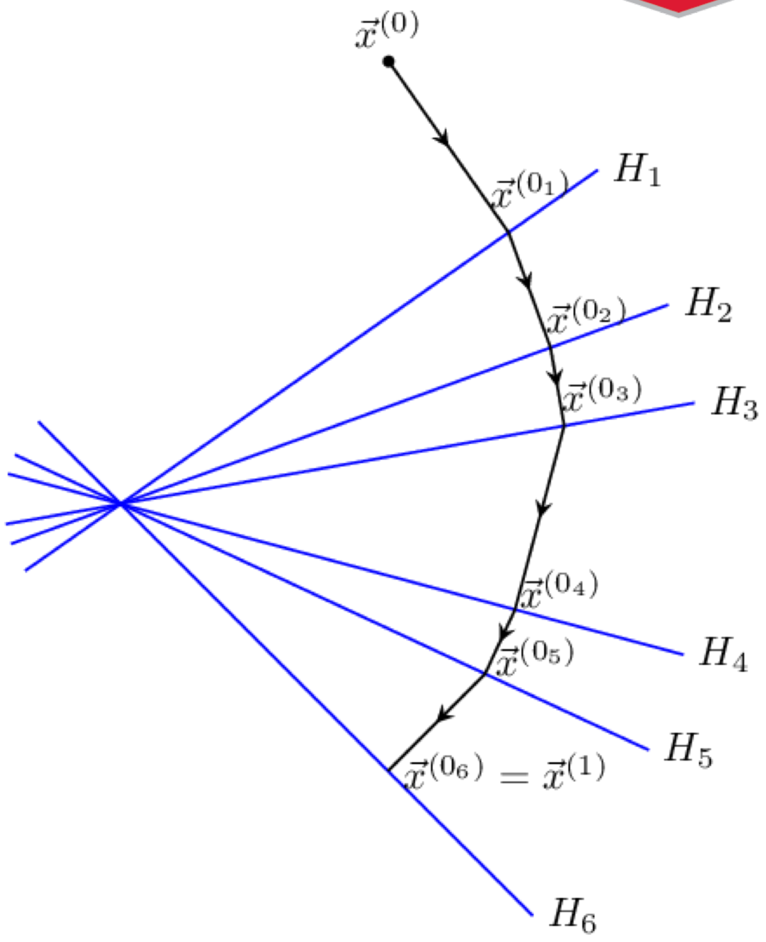


Image Reconstruction

- Convert the energy loss to water equivalent path length (WEPL) through the object, then reconstruct the RSP in each voxel
- Preprocessing software by Robert Johnson to cut protons undergoing large angle scatters
- Feldkamp-Davis-Kress (FDK) conebeam filtered backprojection algorithm
  - Developed for xCT with straight-line paths and adapted to pCT
  - Starting solution for iterative reconstruction
- Most likely path (MLP) algorithm by Erdelyi approximates the curved path through the object
  - Similar to the Penfold algorithm
  - Penfold pulls log term out of integral, Erdelyi calculates directly

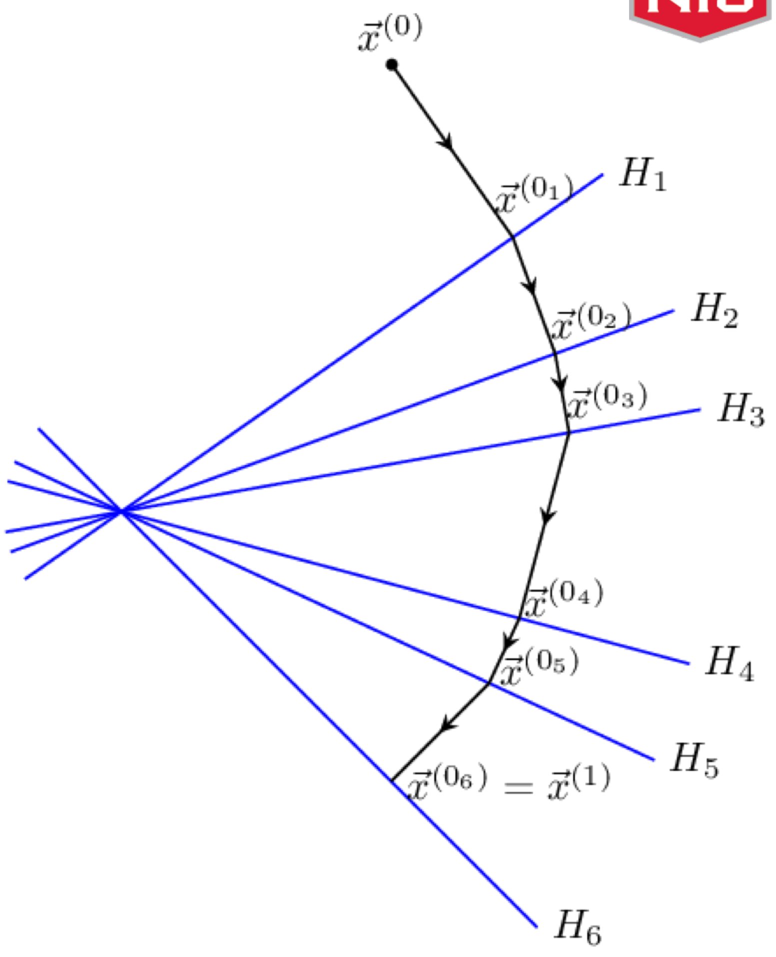
Iterative Reconstruction Methods

- Measure the WEPL of each proton history i
  - $\text{WEPL}_i = \sum_j d\ell_{i,j}\text{RSP}_{i,j} \rightarrow b_i = \vec{a}_i \cdot \vec{x}$
- Sparse matrix equation of the form $\boldsymbol{A}\vec{x} = \vec{b}$, where $\vec{x}$=RSP is unknown
  - $\boldsymbol{A}$: sparse matrix of size number of protons by number of image voxels. Each row in the matrix $\vec{a}_i$ contains an approximate chord length of that proton's path through each of the voxels.
  - $\vec{x}$: Column vector of the RSP in each voxel, updates with each iteration
  - $\vec{b}$: Column vector of the WEPL for each proton
  - There are many more proton histories than image voxels, 120 mil x 3 mil for a 6 minute scan.
  - Each proton passes through a limited number of voxels, meaning many of the elements in $\boldsymbol{A}$ are zero
- Each proton history represents a hyperplane H_i, where the intersection point is the solution.
- Starting with some initial solution $\vec{x}^{(0)}$, we project the solution onto the first hyperplane, then continue to sequentially project the solution from one hyperplane to the next until we get to the last hyperplane.



Parameter Choice in Iterative Algorithms

- Noise in the system changes the intersection “point” of the hyperplanes to an intersection “region”
- This means that the choice of parameters can be influential on the final solution.
- Some parameters are chosen for practical reasons, the remaining parameters are chosen to optimize the image.



Pixel size: 0.8 mm	Number of FDK projections: 180
Slice thickness: 1.25 mm	Reconstruction volume diameter: 24 cm
Maximum number of iterations: 20	Number of blocks/strings: 40

- We optimize the relaxation parameter λ for 5 different algorithms and 3 different imaging doses.

Image Comparisons

- We reconstructed the same data set with:
  - Five combinations of reconstruction algorithms: CARP, CARP+TVSi, DROP, DROP+TVSi, DROP+TVSb
  - 25 relaxation parameters ranging from 0.002 to 1.8
  - FDK + 20 iterations
  - 3 dose levels: 3 min, 6 min, 12 min
    - Note: dose, length of scan, and number of proton histories are related and will be used interchangeably
    - 6 min = 1.02 mGy = 120 million protons in 3×10^{6} voxels.
- Calculated the cost function to find the best image for each reconstruction algorithm and dose level