Correcting Detector Plane Misalignment with Projective Geometry

8th Annual Loma Linda Algorithm Workshop

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ProtonVDA

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Projective Geometry - History

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- 19th century - organize geometry - Felix Klein - Erlangen program

Projective Geometry

- Projective Plane (2D) - Geometric plane plus line at infinity

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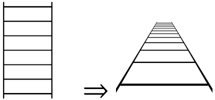

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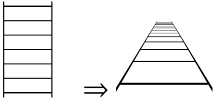
- distance not preserved



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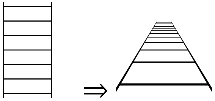

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- distance not preserved
- angles not preserved

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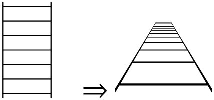

- distance not preserved
- angles not preserved
- no circles, but conic(s)

Projective Transformations

In n dimensions there exists a projective transformation to take any arbitrary (non-collinear) $(n + 2)$ points (projective frame) to any other arbitrary projective frame.

Homogeneous Coordinates

- n dimensional point represented by $n+1$ coordinate values. 2D — $[x \quad y \quad w]$.

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$$[3 \quad 2 \quad 1]$$
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all represent the same point

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$$\begin{matrix}[3 & 2 & 1] \\ [6 & 4 & 2] \\ [0.3 & 0.2 & 0.2] \\ [30 & 20 & 10]\end{matrix}$$

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- Cartesian to homogenous conversion

$$(x, y) \Rightarrow [x \quad y \quad 1]$$

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all represent the same point

- Cartesian to homogenous conversion

$$(x, y) \Rightarrow [x \quad y \quad 1]$$

- Homogeneous to cartesian conversion (normalization)

$$[x \quad y \quad w] \Rightarrow (x/w, y/w)$$

- Point at infinity has $w = 0$.

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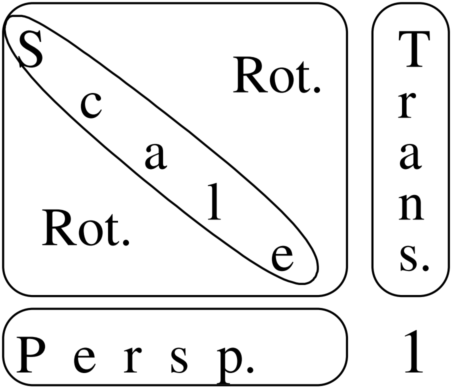
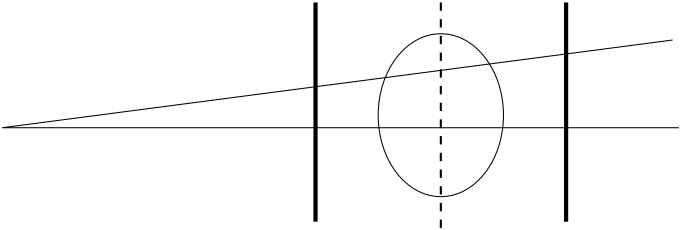
- Transformation composition through matrix multiplication

Projective Transformations

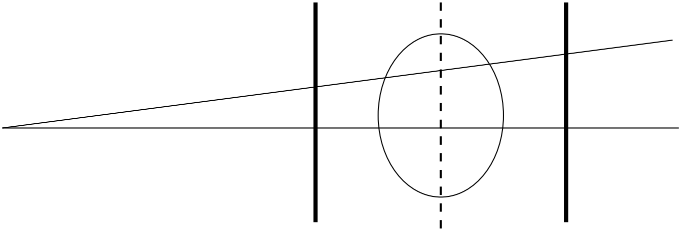


pCT



Assumptions:

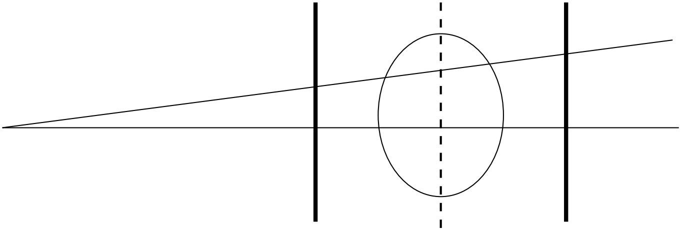
pCT



Assumptions:

- Detector planes perpendicular to beam axis

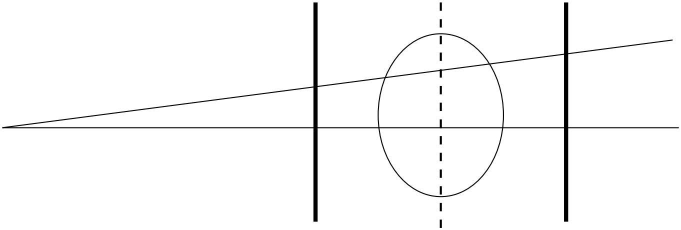
pCT



Assumptions:

- Detector planes perpendicular to beam axis
- Distance of detector planes from isocenter

pCT



Assumptions:

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- Alignment of detector plane axes with beam transverse axes

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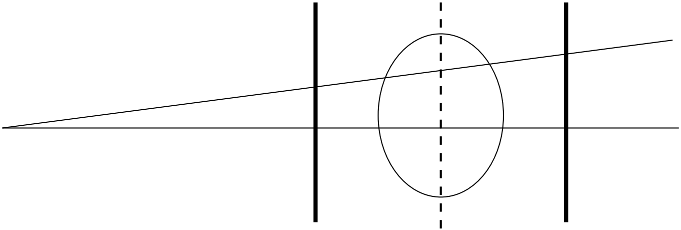

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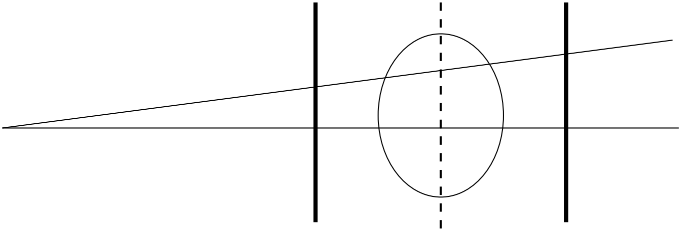

Assumptions:

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- Detector construction

Error Correction

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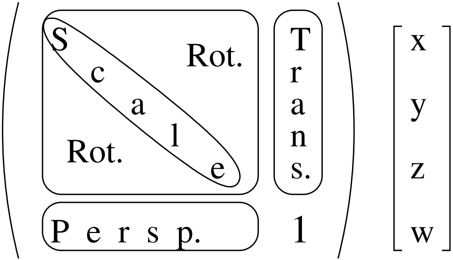
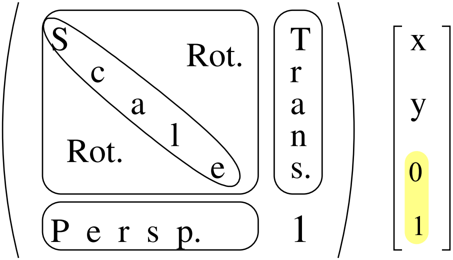
- Idea 1: Combine these affine corrections and perspective projection into single projective transformation matrix.
- Idea 2: Go to 2D
- Idea 3: Directly determine final transformation matrix

2D transformation suffices



2D TRANSFORMATION SUFFICES



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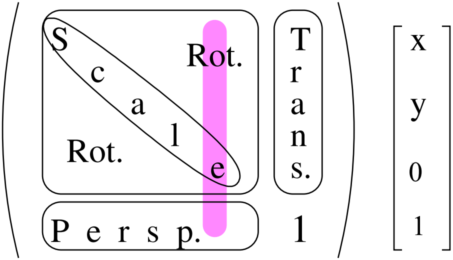

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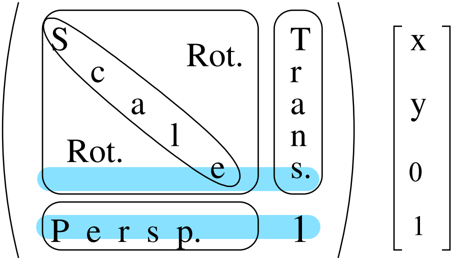

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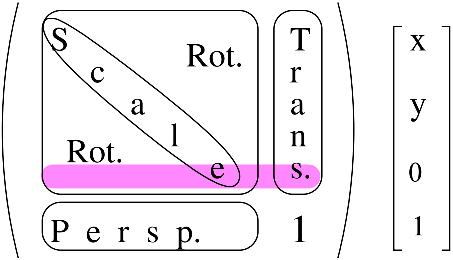

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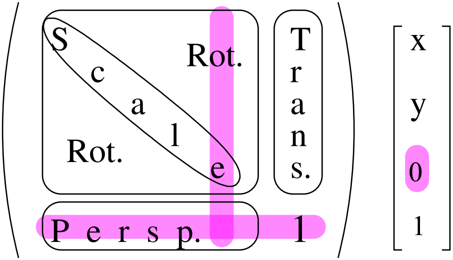


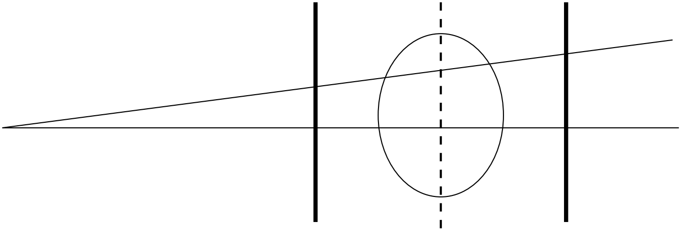
Solving for the matrix

Goal: Find 2D (3x3) projective transformation matrix M that will transform a measured point p_m on a detector into the coordinates of a point p_i on the ideal detector plane.

$$p_i = Mp_m, \begin{pmatrix} x_i \\ y_i \\ w_i \end{pmatrix} = \begin{pmatrix} m_{00} & m_{01} & m_{02} \\ m_{10} & m_{11} & m_{12} \\ m_{20} & m_{21} & 1 \end{pmatrix} \begin{pmatrix} x_m \\ y_m \\ 1 \end{pmatrix}$$

m_{22} set to 1.
8 remaining parameters to find.

Ideal and Measured Points



- Spot mean position

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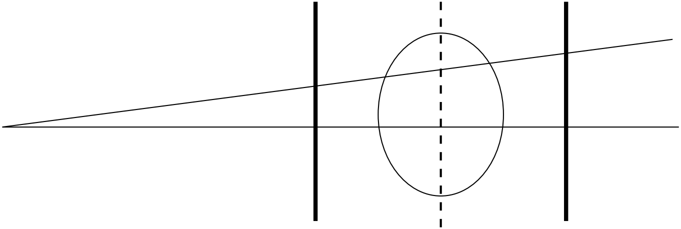

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- Accelerator plan spot position on isocenter plane projected to ideal detector plane

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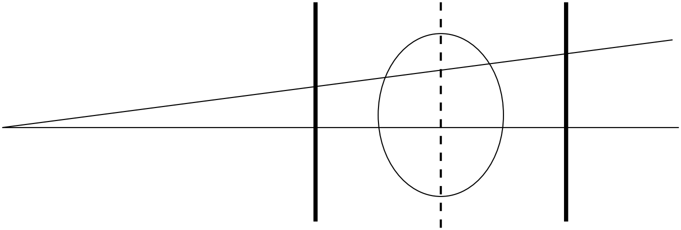

- Spot mean position
- Accelerator plan spot position on isocenter plane projected to ideal detector plane
- Mean of measured hits of spot as measured point

Solving for M

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- Go to overdetermined system

Solving for M

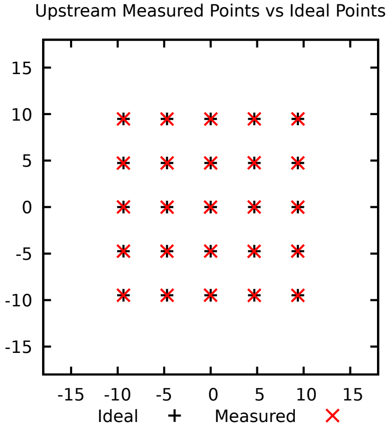
Solve using non-linear optimization (modified Levenberg-Marquardt).
Given k measured points p_m and their corresponding ideal representations(p_m, p_i) point pairs, create $2k$ expressions to minimize,

$$\frac{x(Mp_{m_j})}{w(Mp_{m_j})} - x_{i_j}$$
$$\frac{y(Mp_{m_j})}{w(Mp_{m_j})} - y_{i_j}$$

or

$$\frac{m_{00}x_{m_j}+m_{01}y_{m_j}+m_{02}}{m_{20}x_{m_j}+m_{21}y_{m_j}+1} - x_{i_j}$$
$$\frac{m_{10}x_{m_j}+m_{11}y_{m_j}+m_{12}}{m_{20}x_{m_j}+m_{21}y_{m_j}+1} - y_{i_j}$$

Upstream detector — base case



Computed:

$$\begin{pmatrix} 0.999906 & -0.00058756 & 0.003631 \\ -0.0002755 & 0.999784 & 0.0005824 \\ 4.8408E\text{-}05 & 1.3831E\text{-}06 & 1.0 \end{pmatrix}$$

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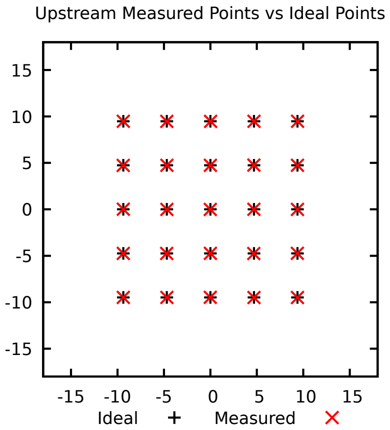

Ideal:

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Upstream detector — base case

Figure: Uncorrected

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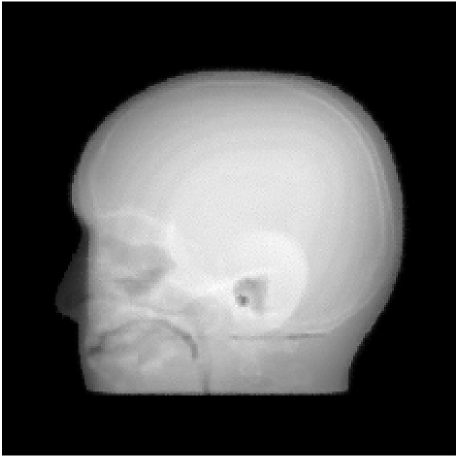
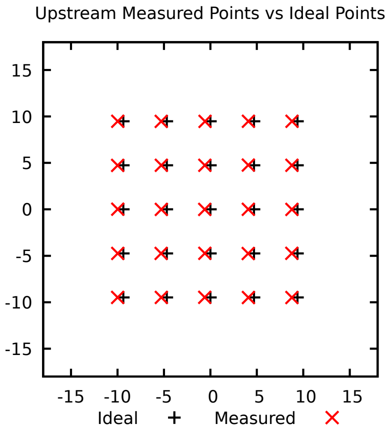


Figure: Corrected

Upstream detector — 6mm transverse translation



Computed:

$$\begin{pmatrix} 0.999877 & -0.00058758 & 0.603557 \\ -0.00027545 & 0.999755 & 0.0004168 \\ 4.8403\text{E-}05 & 1.3714\text{E-}06 & 1.0 \end{pmatrix}$$

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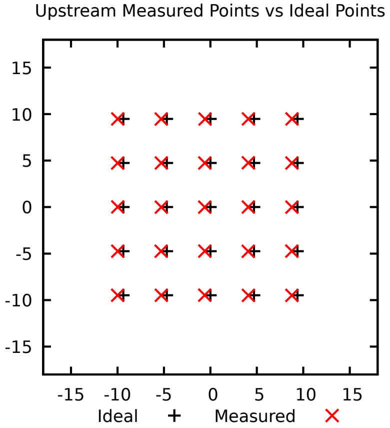

Ideal:

$$\begin{pmatrix} 1.0000000 & 0.00000000 & T_x \\ 0.0000000 & 1.00000000 & T_y \\ 0.0000000 & 0.00000000 & 1.0 \end{pmatrix}$$

Computed:

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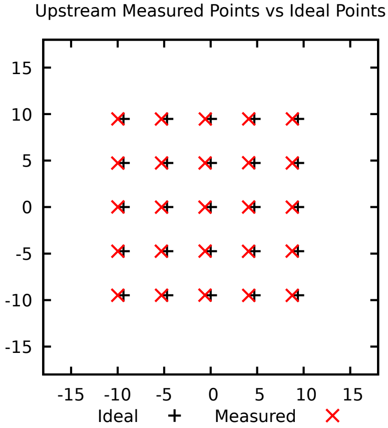

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$$T_1 = 6.03557 \text{ mm},$$
$$T_2 = 0.603557/0.999877 = 6.03631 \text{ mm}$$

Upstream detector — 6mm transverse translation

Figure: Uncorrected

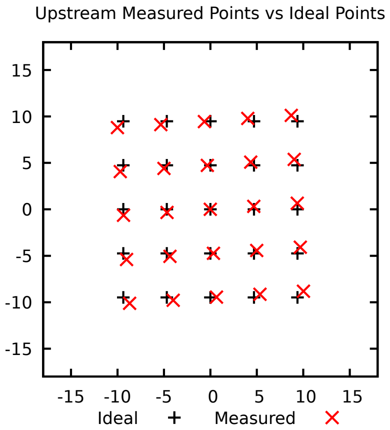
Upstream detector — 6mm transverse translation

Figure: Corrected

Upstream detector — 6mm transverse translation

Figure: Baseline

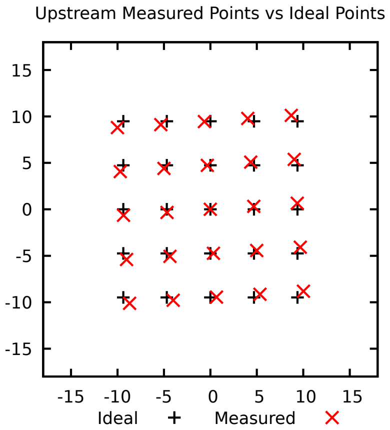
Upstream detector — 4° axial rotation



Computed:

$$\begin{pmatrix} 0.997589 & 0.069128 & 0.003283 \\ -0.069904 & 0.997270 & 0.003206 \\ 0.000050 & 0.000030 & 1.0 \end{pmatrix}$$

Upstream detector — 4° axial rotation



Ideal:

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0.0 \\ \sin\theta & \cos\theta & 0.0 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$$

Computed:

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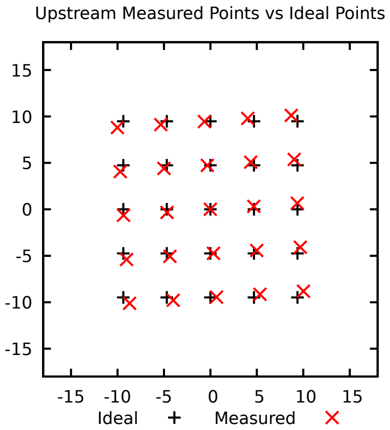

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$$R_1 = \cos^{-1}(0.997589) \Rightarrow \theta = \pm 3.979^\circ$$
$$R_2 = \cos^{-1}(0.997270) \Rightarrow \theta = \pm 4.234^\circ$$
$$R_3 = \sin^{-1}(-0.069904) \Rightarrow \theta = -4.008^\circ$$
$$R_4 = \sin^{-1}(-0.069128) \Rightarrow \theta = -3.964^\circ$$

Upstream detector — 4° axial rotation

Figure: Uncorrected

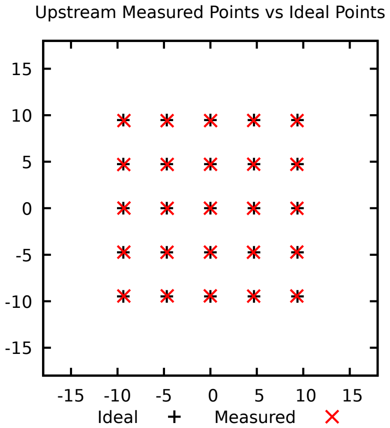
Upstream detector — 4 deg axial rotation

Figure: Corrected

Upstream detector — 4 deg axial rotation

Figure: Baseline

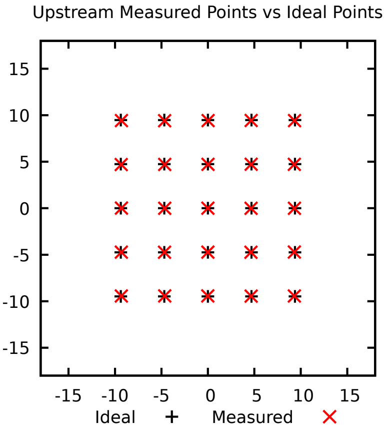
Upstream detector — 12mm axial translation



Computed:

$$\begin{pmatrix} 1.006587 & -0.000506 & 0.003253 \\ -0.000133 & 1.005249 & 0.003159 \\ 0.000051 & 0.000018 & 1.0 \end{pmatrix}$$

Upstream detector — 12mm axial translation



$$S'_x = S_x + z_u - dz_u/2$$
$$S'_y = S_y + z_u + dz_u/2$$
$$k_x = S'_x/(S'_x + \Delta z)$$
$$k_y = S'_y/(S'_y + \Delta z)$$

Ideal:

$$\begin{pmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Computed:

$$\begin{pmatrix} 1.006587 & -0.000506 & 0.003253 \\ -0.000133 & 1.005249 & 0.003159 \\ 0.000051 & 0.000018 & 1.0 \end{pmatrix}$$

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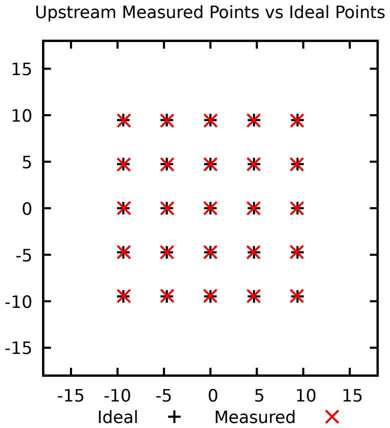

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$$k_x \Rightarrow \Delta z = \text{-11.871 mm}$$
$$k_y \Rightarrow \Delta z = \text{-11.404 mm}$$

Upstream detector — 12mm axial translation

Figure: Uncorrected

Upstream detector — 12mm axial translation

Figure: Corrected

Upstream detector — 12mm axial translation

Figure: Baseline

Conclusion and Future Work

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- Feasible detector alignment correction

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- Alignment using image data instead of alignment scan

Acknolwedgements

- NIU Computer Science Department
    - Nicholas Karonis, Cesar Ordoñez, John Winans
- NIU Physics Department
    - George Coutrakon
- ProtonVDA
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- Northwestern Medicine Proton Center
    - James Welsh, Mark Pankuch