Time-of-flight proton CT

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Physics in Medicine & Biology

ACCEPTED MANUSCRIPT

Relative stopping power resolution in time-of-flight proton CT

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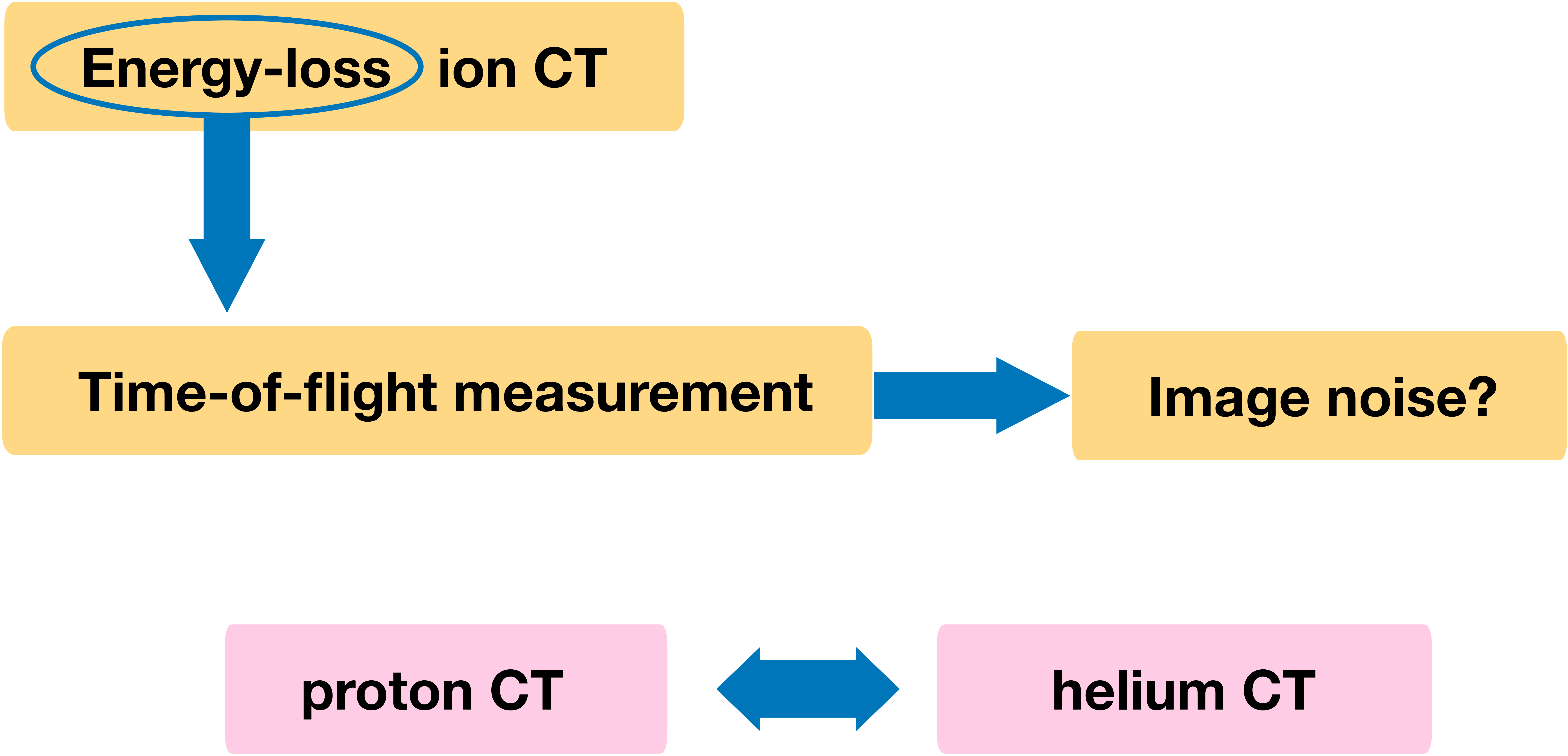
What is an Accepted Manuscript?

Accepted Manuscript PDF

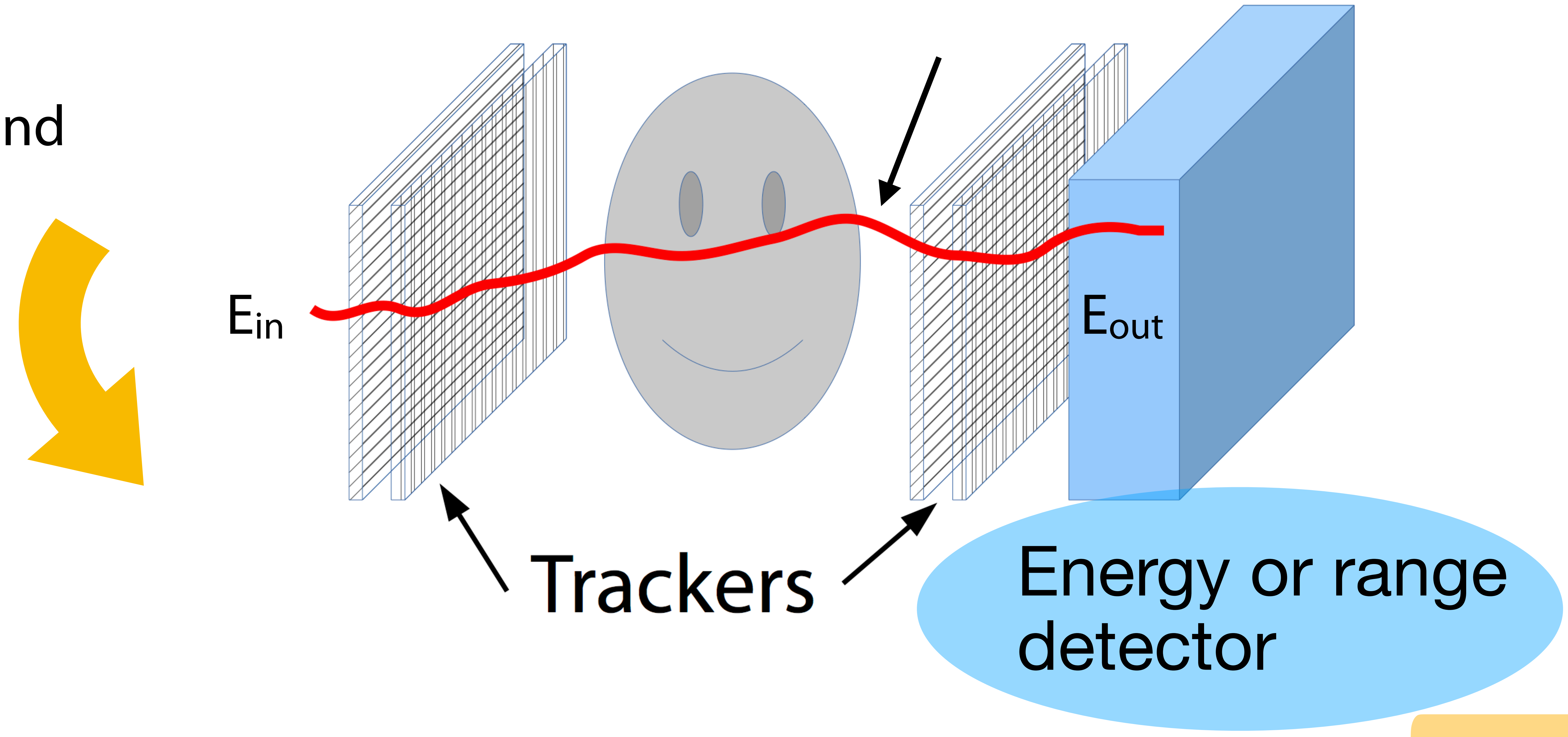
DOI: 10.1088/1361-6560/ac7191

I will speak about …



Typical list-mode ion CT set-up

Rotate around patient



Generates list mode data: Trajectory estimate and water equivalent path length for each proton.

Calorimeter

Range telescope

Time-of-flight

Comprehensive review:

Johnson, R. P. (2018). Review of medical radiography and tomography with proton beams. Reports on Progress in Physics, 81(1), 016701. https://doi.org/10.1088/1361-6633/aa8b1d

Reconstruction problem in ion CT

Estimated ion path across object

Measured by range telescope

or

Measured by calorimeter or time-of-flight detector

$$\int_{\Gamma} \mathrm{RSP}(x)\mathrm{d}x = \mathrm{WEPL}(E_{\mathrm{out}}) = \int_{E_{\mathrm{in}}}^{E_{\mathrm{out}}} 1/S_w(E)\mathrm{d}E$$

Relative stopping power: material property

Water equivalent path length

Stopping power in water

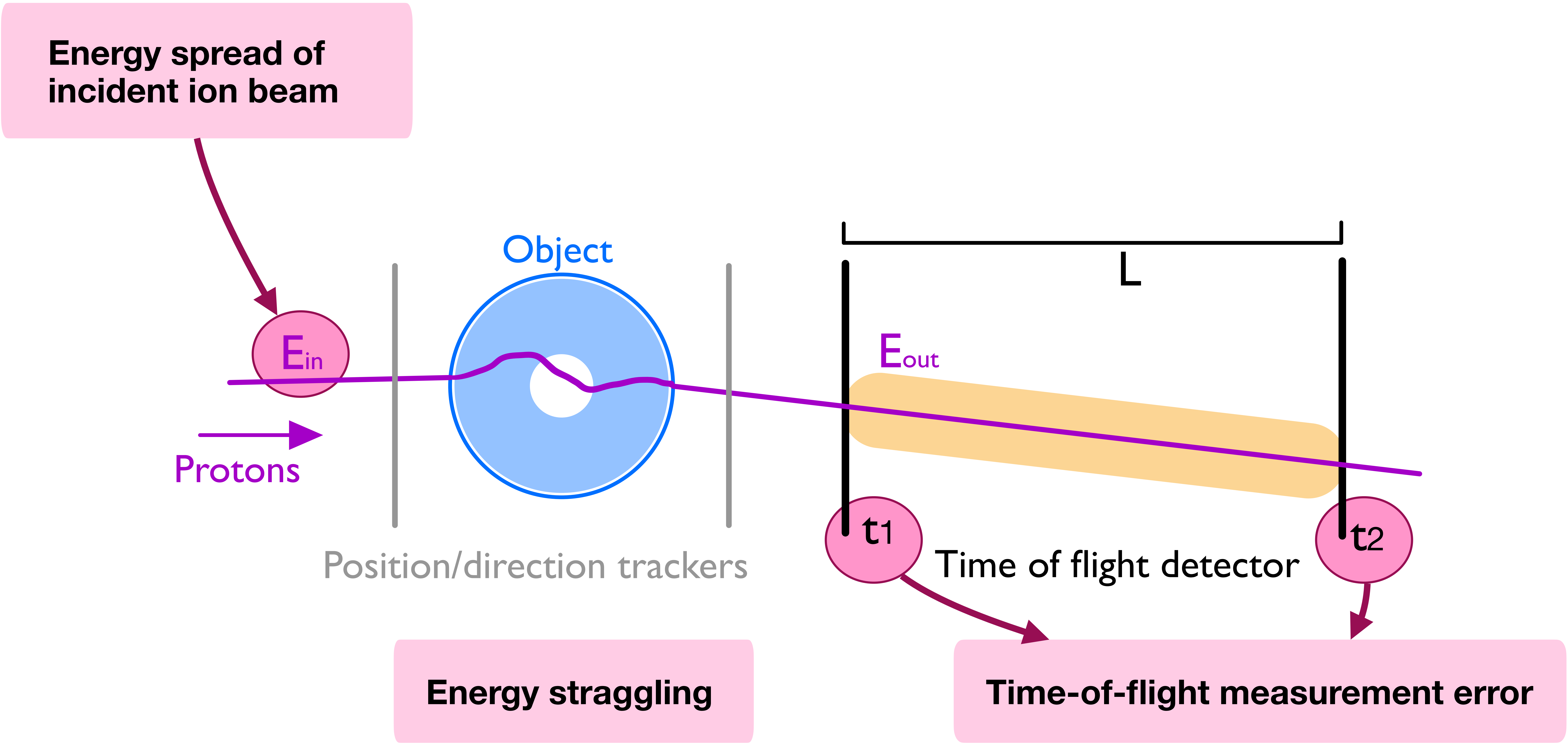
Question to be answered:

What is the impact of energy uncertainty on the estimated RSP map in terms of noise?

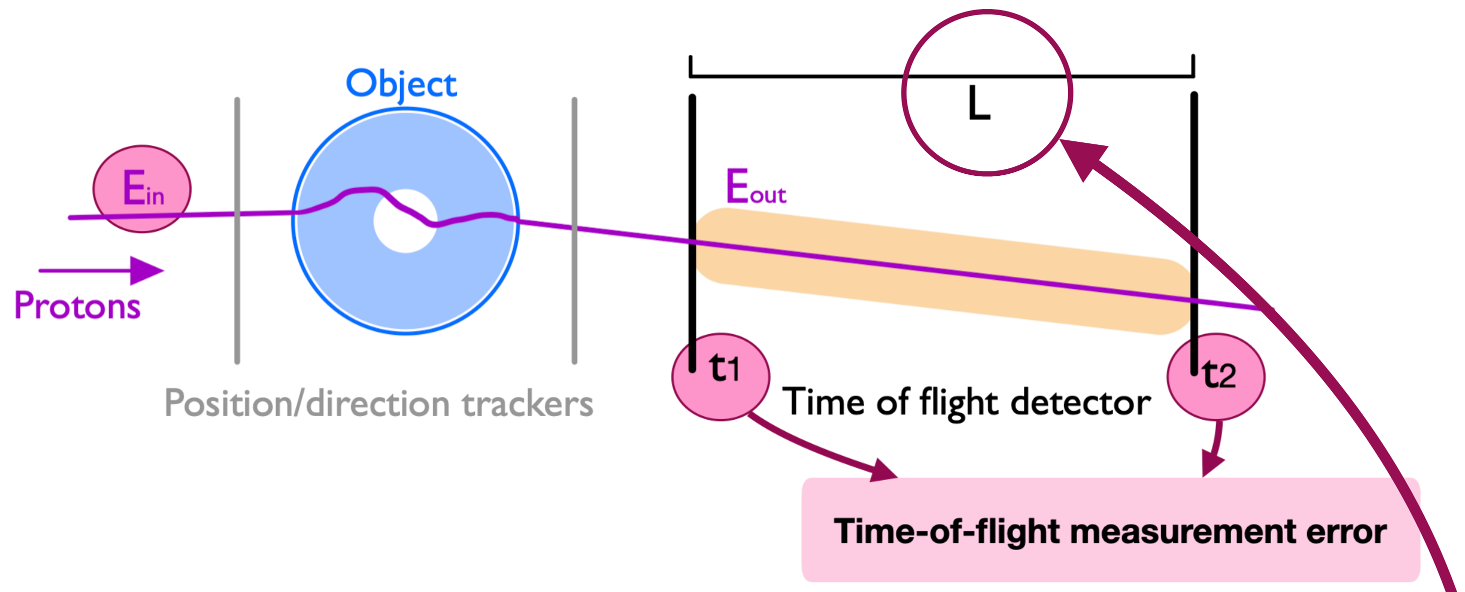
$$\int_{\Gamma} \mathrm{RSP}(x)\,\mathrm{d}x = \mathrm{WEPL}(E_{\mathrm{out}}) = \int_{E_{\mathrm{in}}}^{E_{\mathrm{out}}} 1/S_w(E)\,\mathrm{d}E$$

Error propagation

Sources of energy error/uncertainty



Time-of-flight measurement error



Relativistic energy - velocity relation:

$$E_{\text{out}} = \frac{m_p c^2}{\sqrt{1-(v/c)^2}} - m_p c^2 \quad \text{with} \quad v = \frac{L}{t_2 - t_1},$$

time-of-flight

First order error propagation:

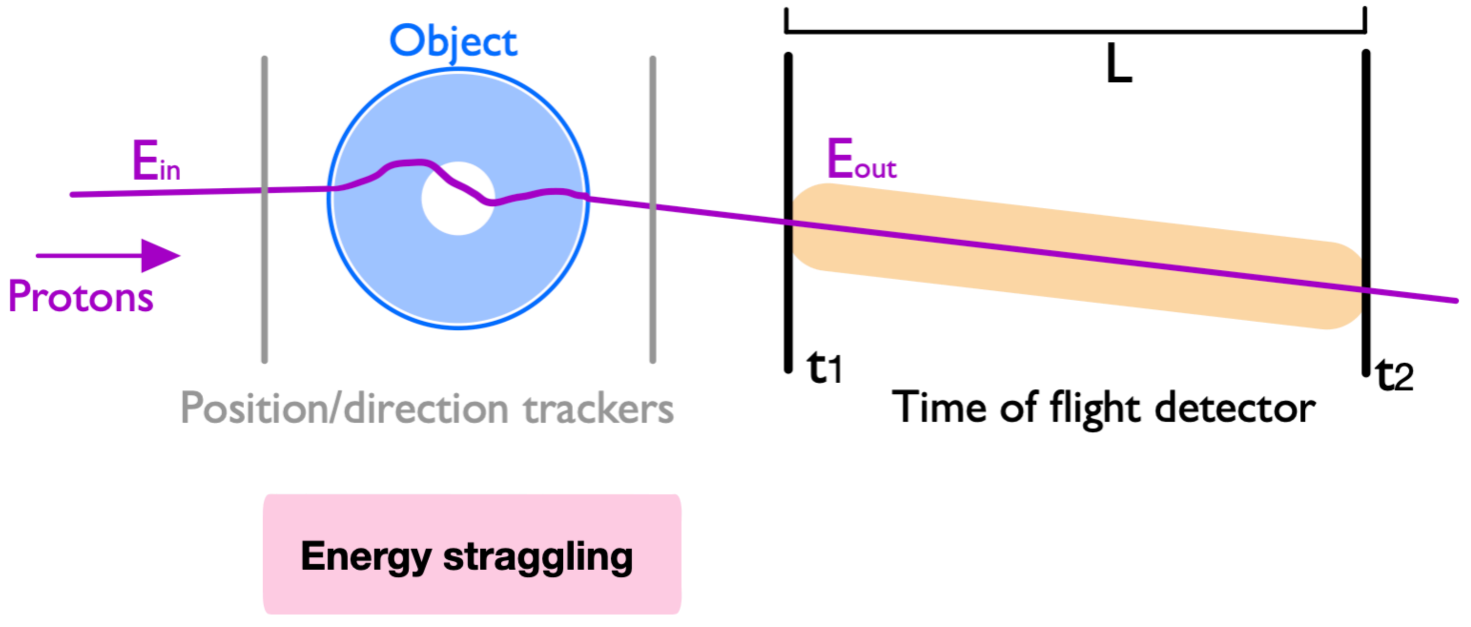
velocity error (variance)

$$\sigma^2_{E_{\text{out}},\text{TOF}}(E_{\text{out}}) = \left|\frac{\mathrm{d}E}{\mathrm{d}t_1}\right|^2 \sigma^2_{t_1} + \left|\frac{\mathrm{d}E}{\mathrm{d}t_2}\right|^2 \sigma^2_{t_2} = \frac{1}{m_p^4 c^6}\left(E^2_{\text{out}} + 2m_p c^2 E_{\text{out}}\right)^3 \frac{\sigma_t^2}{L^2}$$

energy error (variance)

Note: $\sigma^2_{E_{\text{out}},\text{TOF}} \propto E^3_{\text{out}}$

Energy straggling



- **Variation of energy loss within ensemble of ions due to stochastic nature of electromagnetic interactions.**
- **Approximately Gaussian energy distribution.**
- **Variance can be calculated analytically (to first order) [1].**

energy error (variance)

$$\sigma^2_{E_{\text{out}},\text{straggling}}(E_{\text{out}}) = \chi_1^2(E_{\text{out}}) \int_{E_{\text{out}}}^{E_{\text{in}}} \frac{\chi_2(E)}{\chi_1^3(E)} \mathrm{d}E$$

Solve via numerical integration.

$$\chi_1(E) = K \frac{1}{\beta^2} \left[\ln \left(\frac{2m_e c^2 \beta^2}{I(1-\beta^2)} \right) - \beta^2 \right]$$

$$\text{with} \quad \beta = \frac{v}{c} = \left[1 - \left(\frac{m_p c^2}{m_p c^2 + E} \right)^2 \right]^{1/2}$$

$$\chi_2(E) = K m_e c^2 \frac{1 - \beta^2/2}{1 - \beta^2}$$

I: ionisation potential (approx. as water, 75 eV)
m_p: proton mass
m_e: electron mass
K: a constant

[1] Payne, M. G. (1969). Energy Straggling of Heavy Charged Particles in Thick Absorbers. Physical Review, 185(2), 611–623. DOI: 10.1103/PhysRev.185.611

Uncertainty on exit energy

time-of-flight

$$\sigma^2_{E_{\text{out}}}(E_{\text{out}}) = \sigma^2_{E_{\text{out}},\text{straggling}}(E_{\text{out}}) + \sigma^2_{E_{\text{out}},\text{TOF}}(E_{\text{out}})$$

Compare with calorimeter-based ion CT system [1]:

$$\sigma^2_{E_{\text{out}},\text{cal}}(E_{\text{out}}) = \sigma^2_{E_{\text{out}},\text{straggling}}(E_{\text{out}}) + \delta^2 E_{\text{out,cal}} E^2_{\text{out}}$$

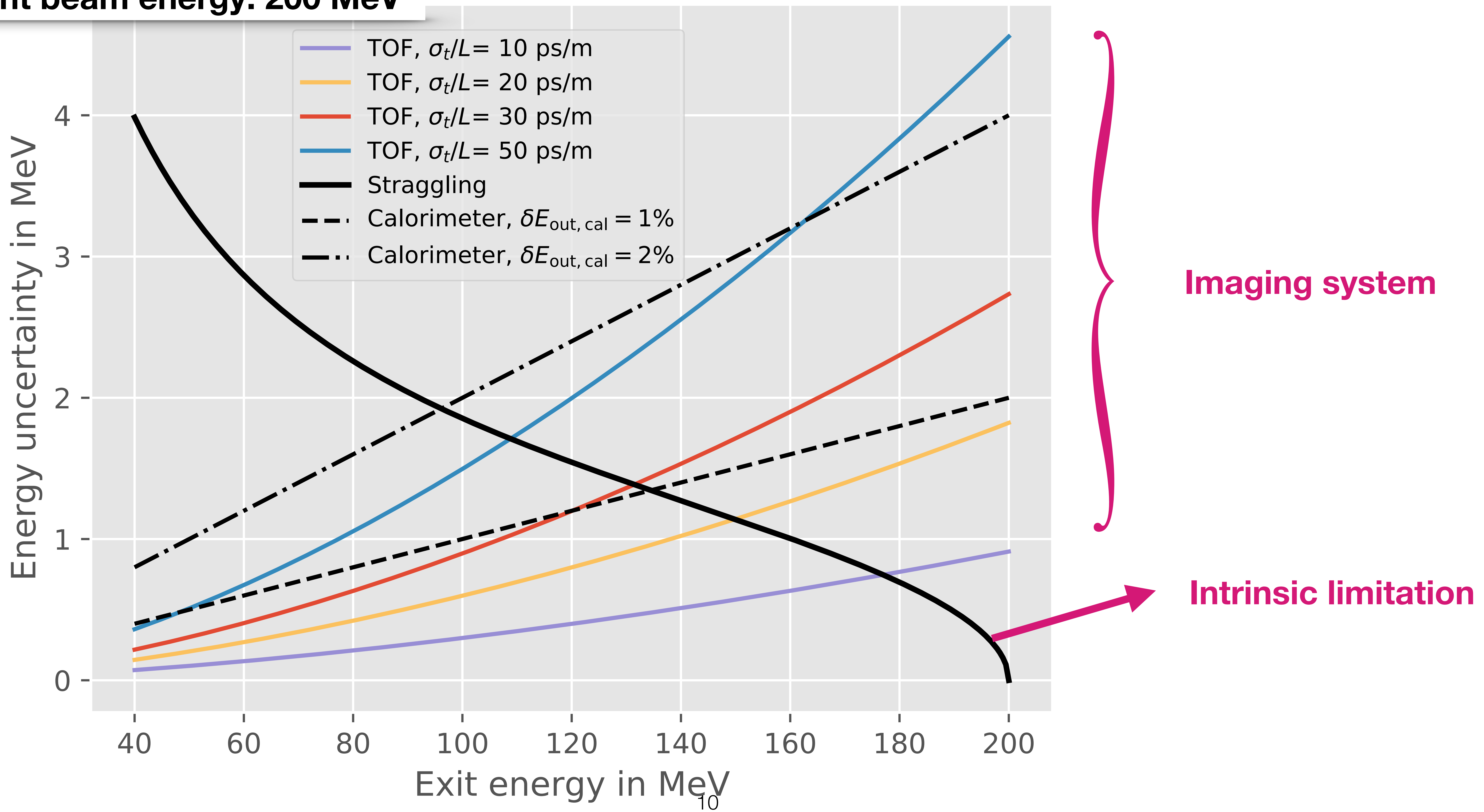
calorimeter

$$\delta E_{\text{out,cal}} \approx 1 - 2\,\%$$

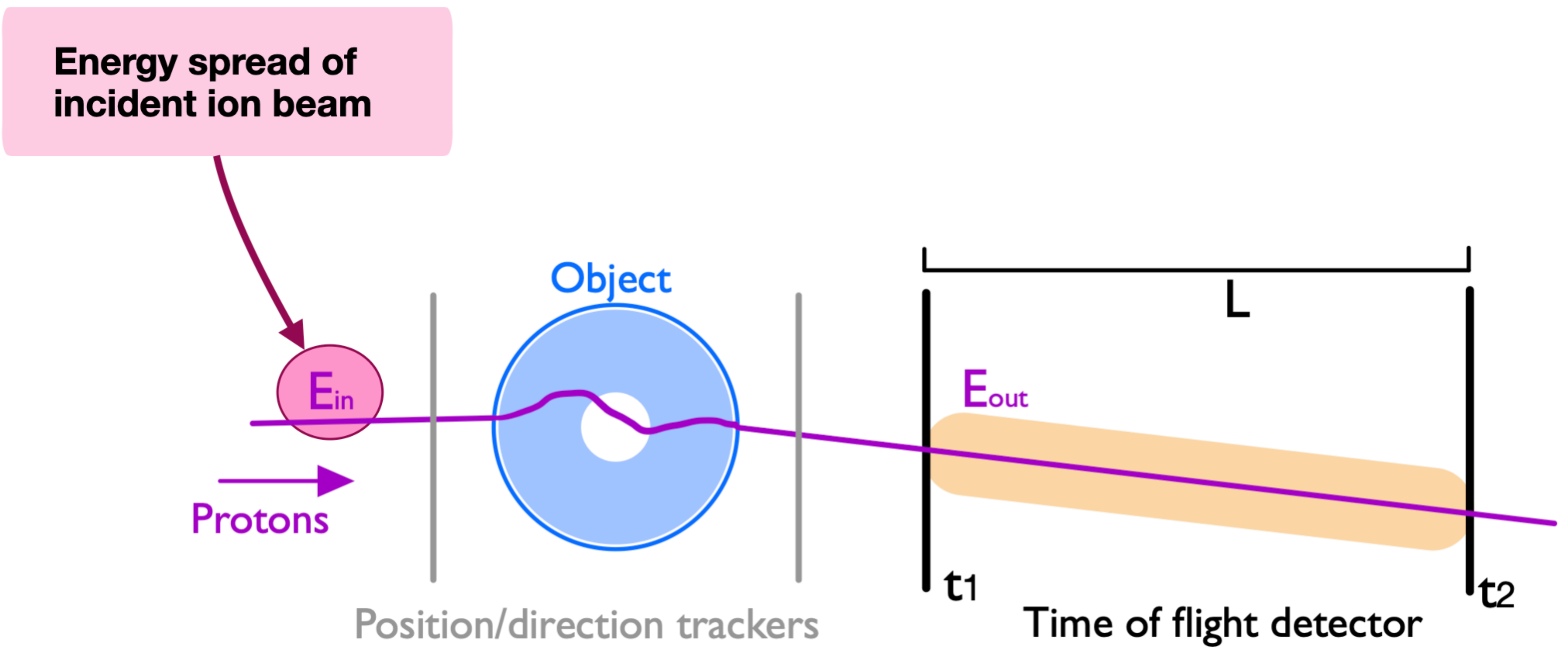
[1] Bashkirov, V. A. et al. (2016). Novel scintillation detector design and performance for proton radiography and computed tomography. Medical Physics, 43(2), 664–674. https://doi.org/10.1118/1.4939255

Energy uncertainty

Incident beam energy: 200 MeV



Energy spread of incident beam



- **Depends on accelerator and beam delivery system.**
- **E.g. synchrotron vs. cyclotron**
- **We assumed 0.5% of beam energy [1].**

$$\sigma^2_{\text{beam}}(E_{\text{in}}) = \delta^2 E_{\text{beam}} E^2_{\text{in}} \quad \text{with} \quad \delta E_{\text{beam}} = 0.5\,\%$$

[1] Schippers, J. M. (2018). Beam Transport Systems for Particle Therapy. In R. Bailey (Ed.), Proceedings of the CAS-CERN Accelerator School: Accelerators for Medical Applications,. Vösendorf, Austria: CERN. https://doi.org/10.23730/CYRSP-2017-001.241

WEPL uncertainty

$$\sigma^2_{\Delta E}(E_{\text{out}}) = \sigma^2_{E_{\text{out}},\text{straggling}}(E_{\text{out}}) + (\delta E_{\text{beam}} E_{\text{in}})^2 + \sigma^2_{E_{\text{out}},\text{TOF}}(E_{\text{out}})$$

+ multiple Coulomb scattering

First order error propagation:

$$\text{WEPL}(E_{\text{out}}) = \int_{E_{\text{in}}}^{E_{\text{out}}} 1/S_w(E)\text{d}E$$

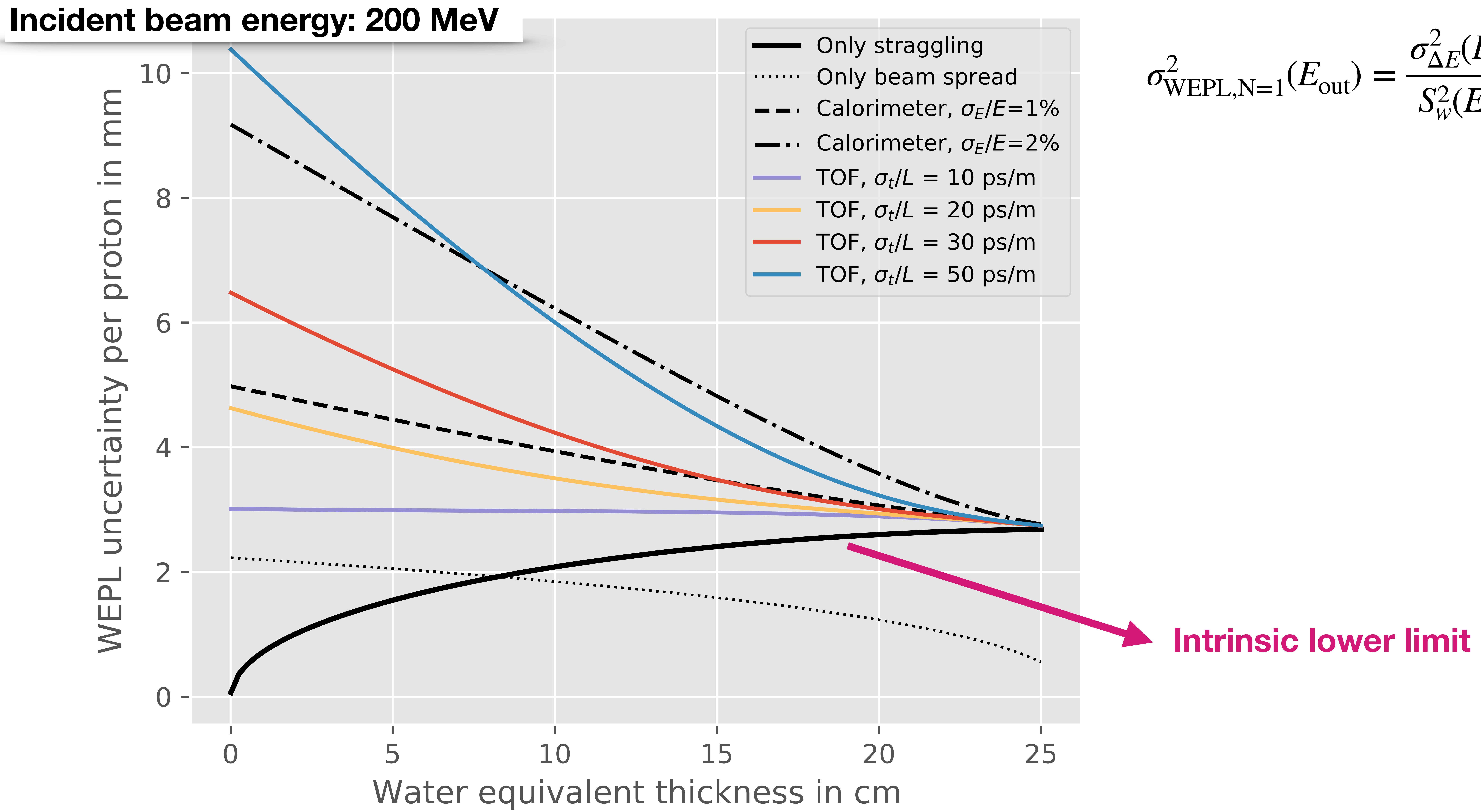
$$\sigma^2_{\text{WEPL}}(E_{\text{out}}) = \frac{\sigma^2_{\Delta E}(E_{\text{out}})}{S_w^2(E_{\text{out}})N} = \frac{\sigma^2_{\Delta E}(E_{\text{out}})}{S_w^2(E_{\text{out}})\,\Phi\,\Delta\xi^2}$$

number of ions

particle fluence (dose)

pixel size, e.g. 1 mm²

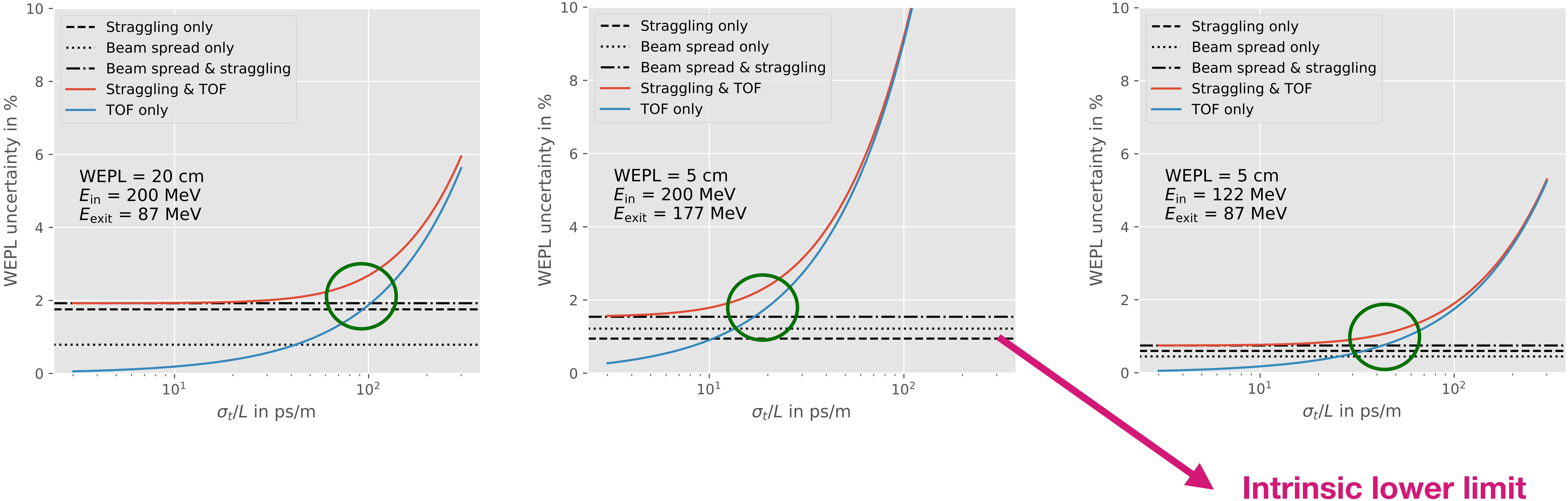
WEPL uncertainty



$$\sigma^2_{\mathrm{WEPL,N=1}}(E_{\mathrm{out}}) = \frac{\sigma^2_{\Delta E}(E_{\mathrm{out}})}{S^2_w(E_{\mathrm{out}})}$$

WEPL uncertainty: TOF vs straggling

$$\sigma^2_{\mathrm{WEPL,N=1}}(E_{\mathrm{out}}) = \frac{\sigma^2_{\Delta E}(E_{\mathrm{out}})}{S^2_w(E_{\mathrm{out}})}$$



Intrinsic lower limit

Ideally: Incident energy should be adjusted as a function of (expected) WEPL (see Stefanie's talk yesterday)

RSP uncertainty via noise reconstruction

Propagate noise from WEPL to RSP: $$\mathrm{WEPL}(E_{\mathrm{out}}) = \int_\Gamma \mathrm{RSP}(x)\,\mathrm{d}x$$

projection images containing WEPL variance → **noise reconstruction [1,2]** → **reconstructed images containing RSP variance**

Assumptions:

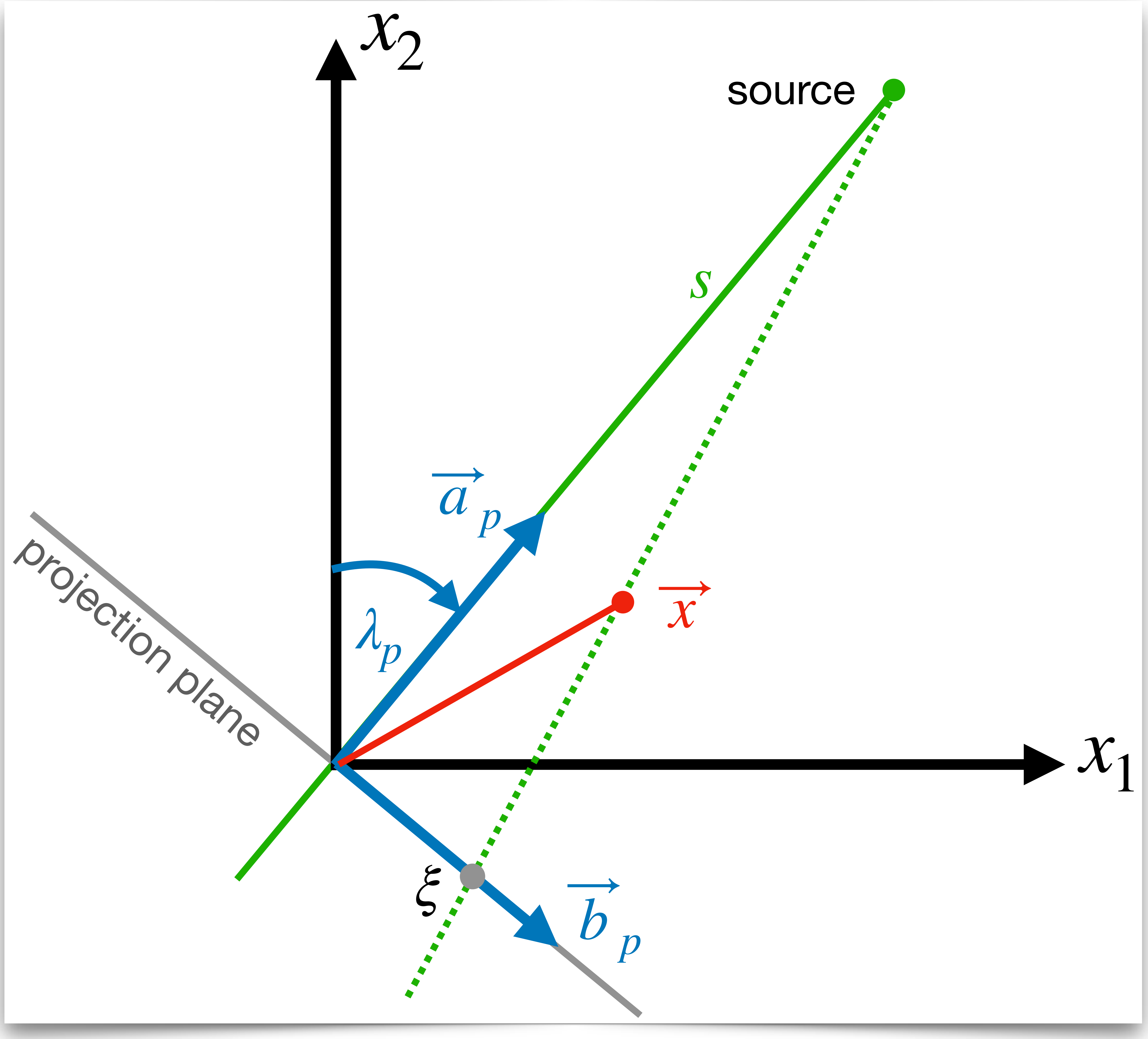
- Ion CT images are reconstructed via filtered backprojection …
- … in fan beam geometry
- Linear interpolation between pixels
- Filtered with an apodized ramp filter

See George's talk this morning

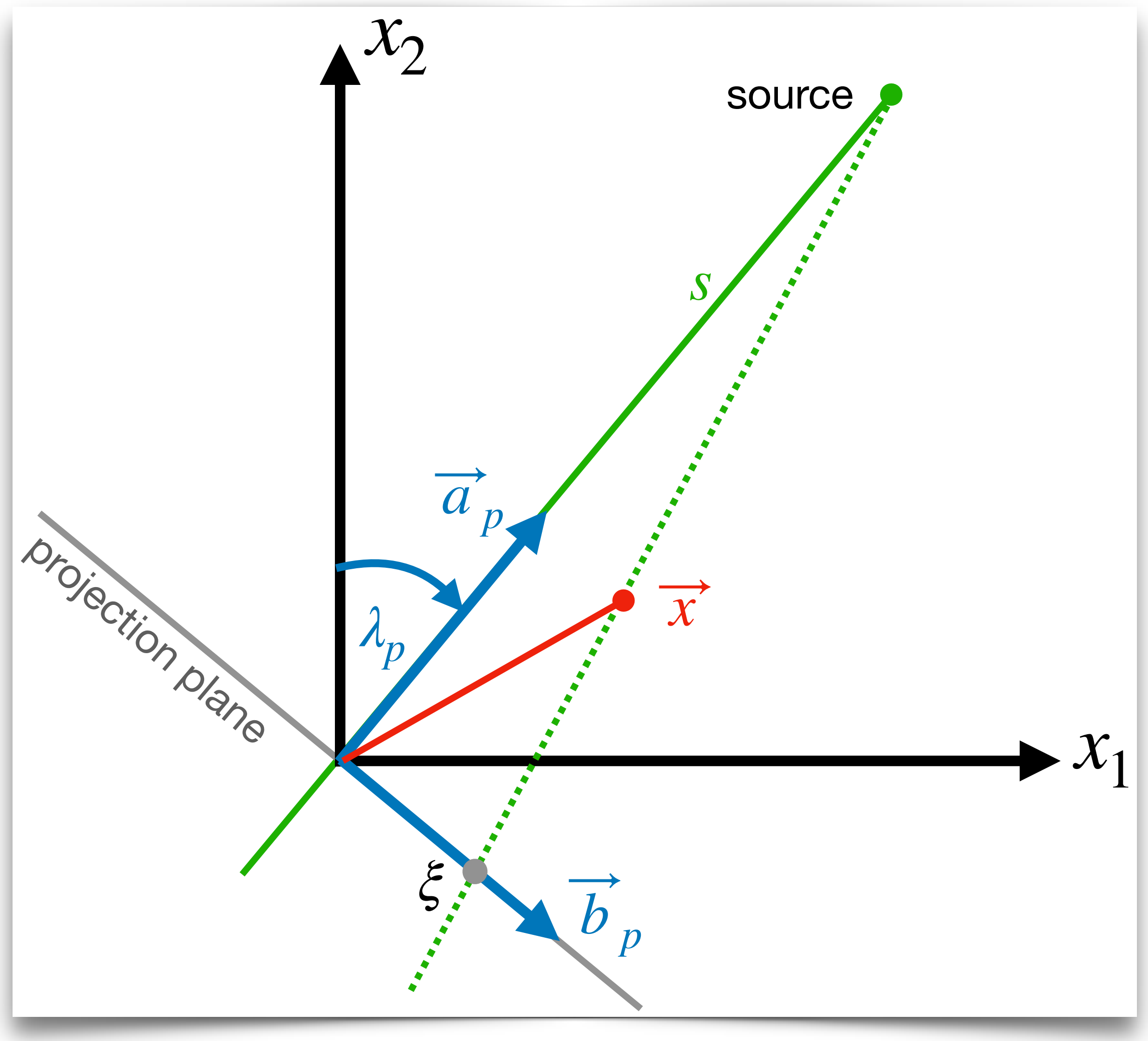
[1] Wunderlich, A., & Noo, F. (2008). Image covariance and lesion detectability in direct fan-beam x-ray computed tomography. Physics in Medicine and Biology, 53(10), 2471–2493. https://doi.org/10.1088/0031-9155/53/10/002

[2] Rädler, M. et al. (2018). Two-dimensional noise reconstruction in proton computed tomography using distance-driven filtered back-projection of simulated projections. Physics in Medicine & Biology, 63(21), 215009. https://doi.org/10.1088/1361-6560/aae5c9

Noise reconstruction: geometry (2D)



RSP uncertainty via noise reconstruction
(approximate)



Backprojection:

$$\mathrm{Var}_{\mathrm{RSP}}(\vec{x}) = f_{\mathrm{interp}} \frac{\Delta\lambda^2}{4} \sum_{p=1}^{P} \left(\frac{\|\vec{a}_{\lambda_p}\|}{\|\vec{x} \cdot (\vec{a}_{\lambda_p}/s) - \vec{a}_{\lambda_p}\|} \right)^4 V_p(\xi_k)$$

approximation

weighting factor

Filtering:

weighting factor

$$V_p(\xi_k) = (\Delta\xi)^2 \sum_{j=-J}^{J-1} h_F^2(\xi_k - \xi_j) \frac{\|\vec{a}_{\lambda_p}\|^2}{\|\vec{a}_{\lambda_p}\|^2 + \xi_j^2} \mathrm{Var}(\lambda_p, \xi_j)$$

Anodized ramp filter:

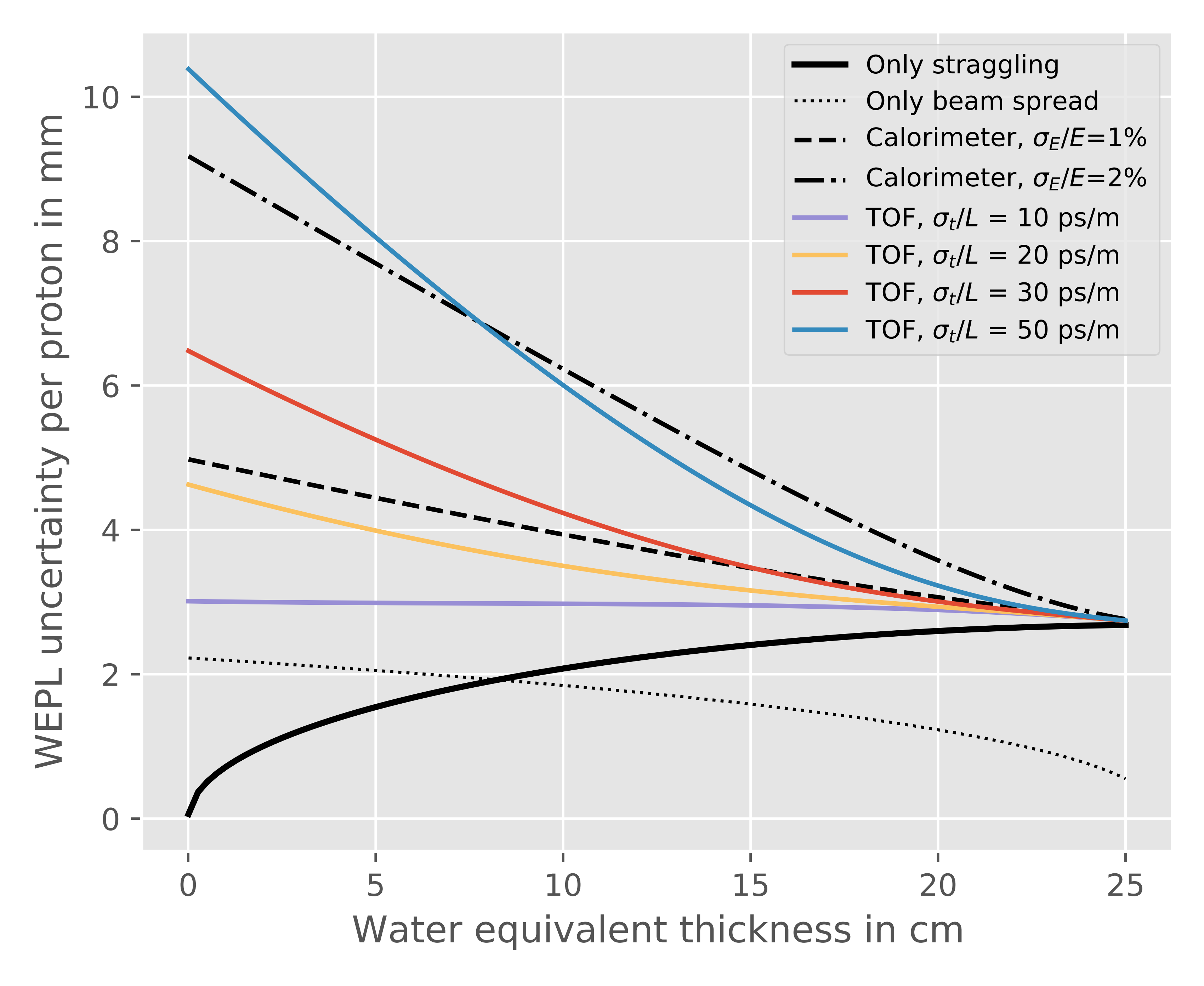
$$h_F(\xi_j) = h_F((j+1/2)\Delta\xi) = \begin{cases} 1/(2\Delta\xi)^2 & \text{for } j = 0, \\ 0 & \text{for } j \text{ even and } j \neq 0 \\ -1/(j\pi\Delta\xi)^2 & \text{for } j \text{ odd,} \end{cases}$$

WEPL variance in pixel j and projection p

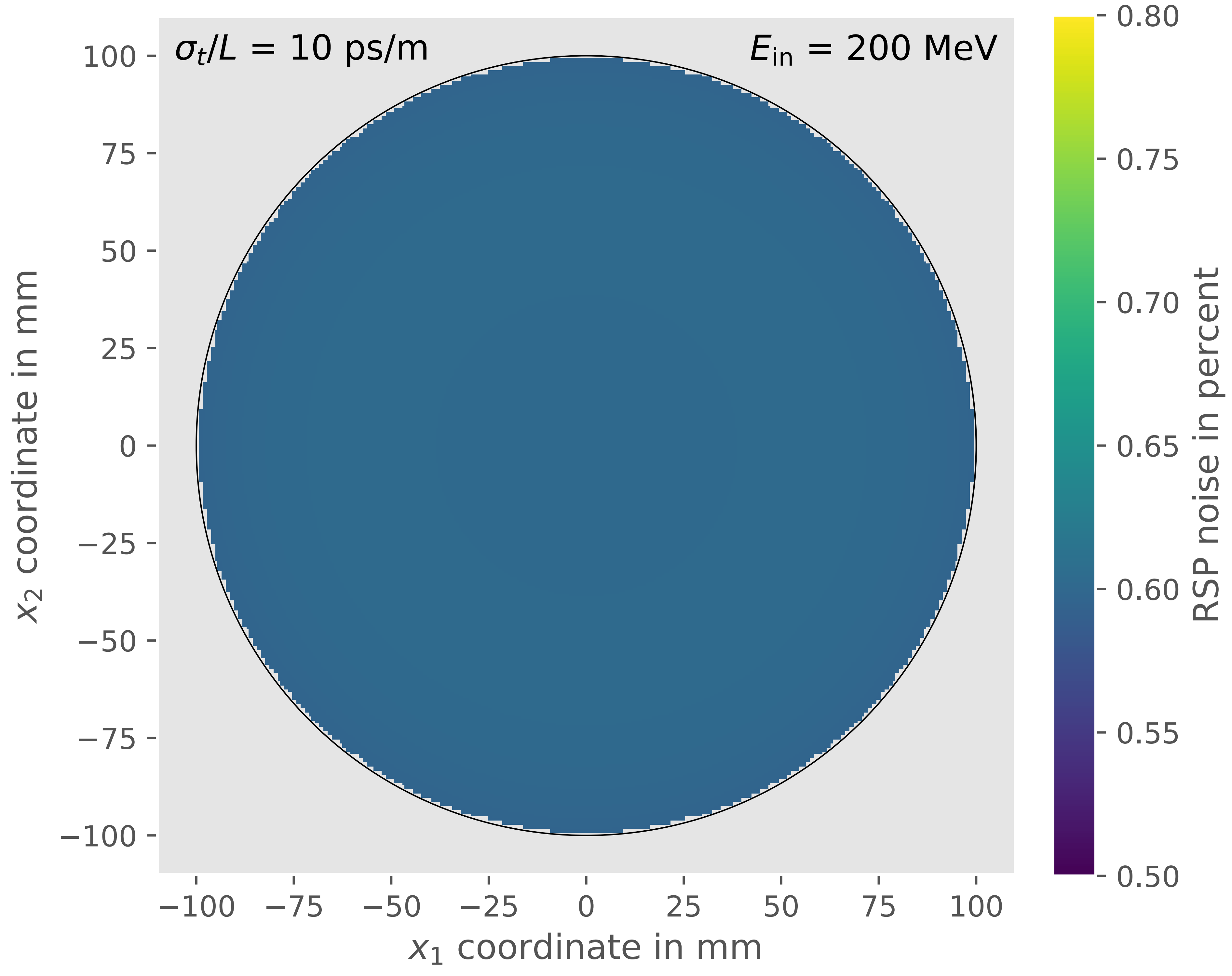
[1] Wunderlich, A., & Noo, F. (2008). Image covariance and lesion detectability in direct fan-beam x-ray computed tomography. Physics in Medicine and Biology, 53(10), 2471–2493. https://doi.org/10.1088/0031-9155/53/10/002

RSP uncertainty in a water cylinder

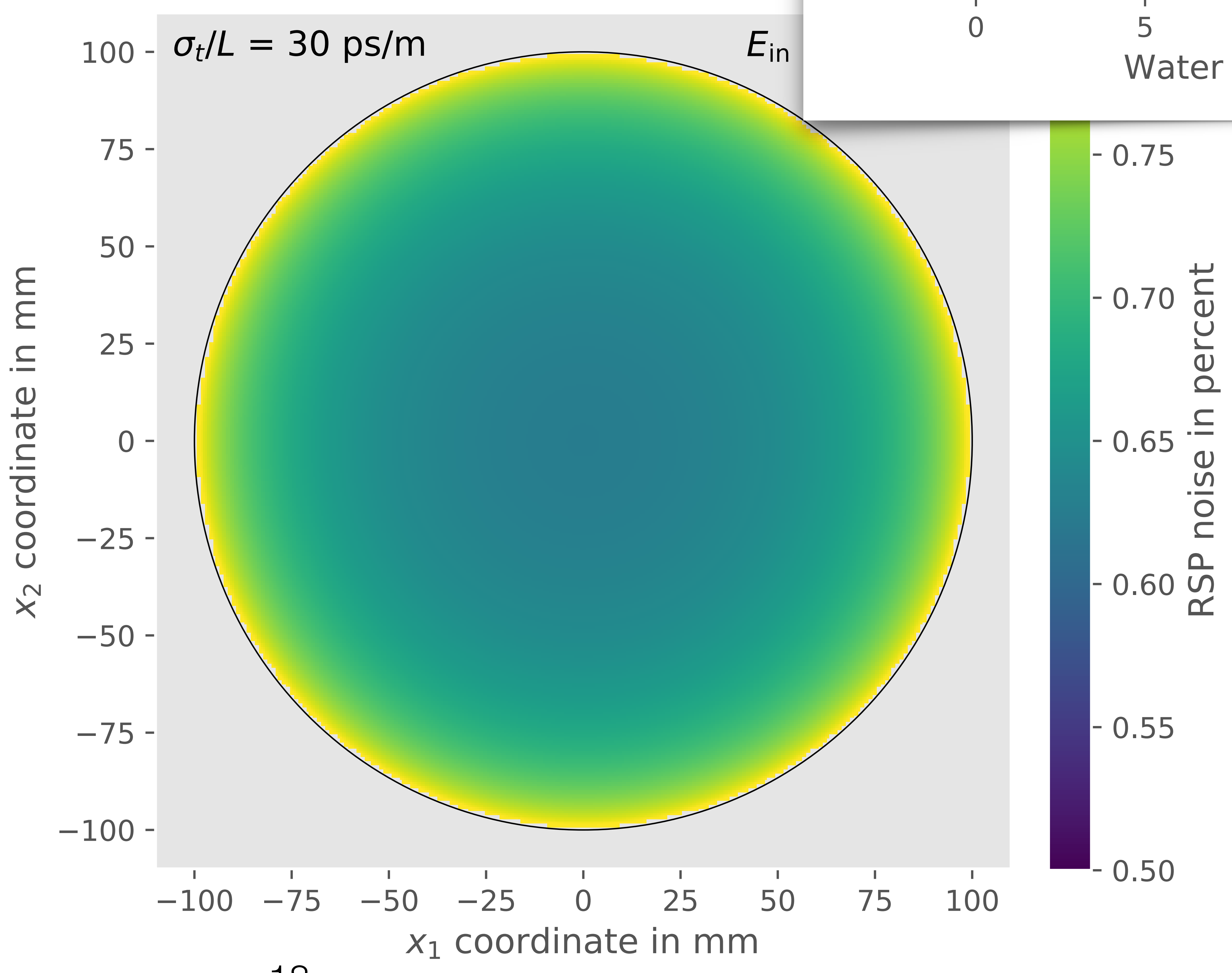
- Diameter: 20 cm
- Dose to center: 10 mGy (full acquisition)
- 1x1 mm^2 pixel size
- Incident beam energy: 200 MeV



velocity error: 10 ps/m



velocity error: 30 ps/m

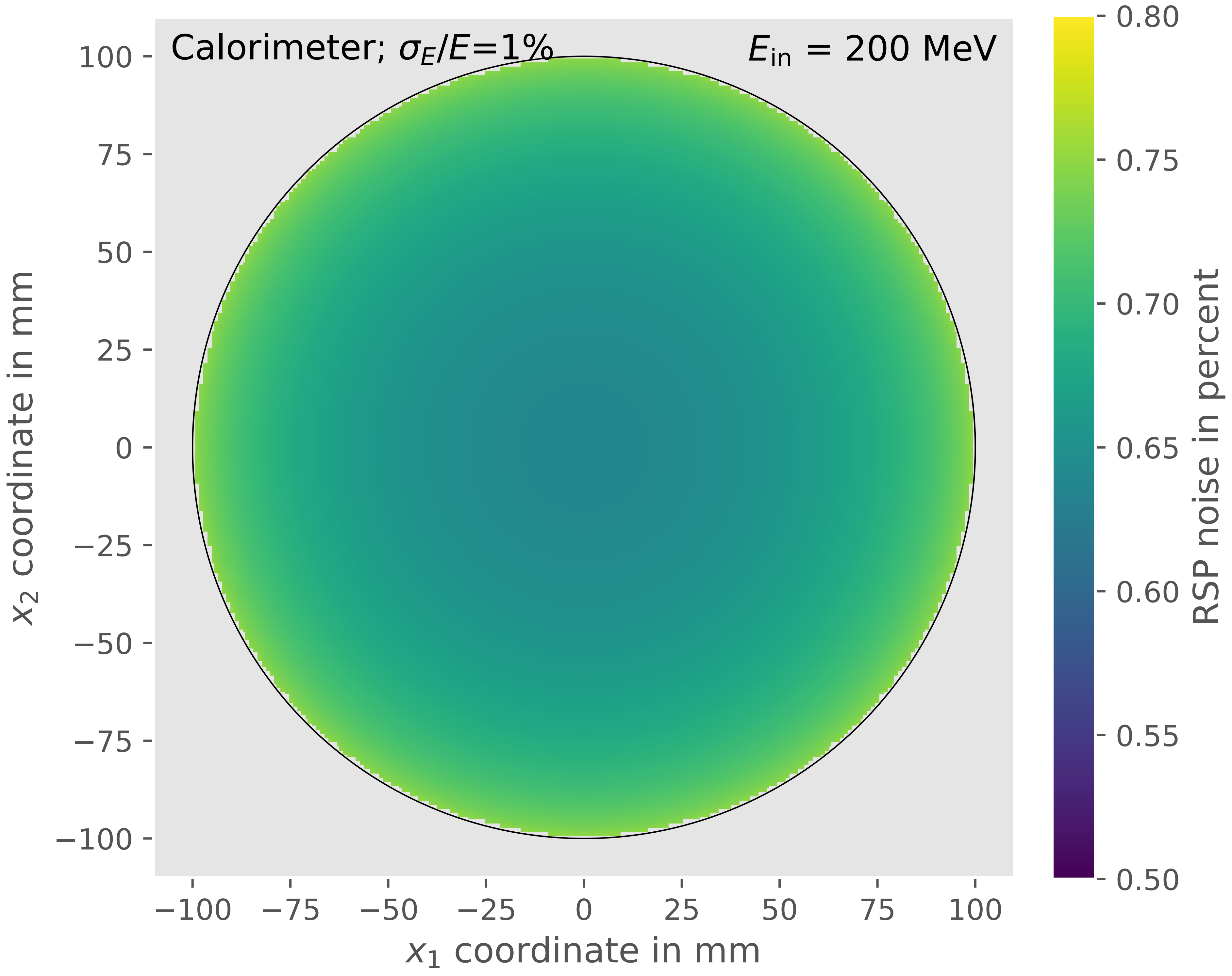


Images noisier towards the edges

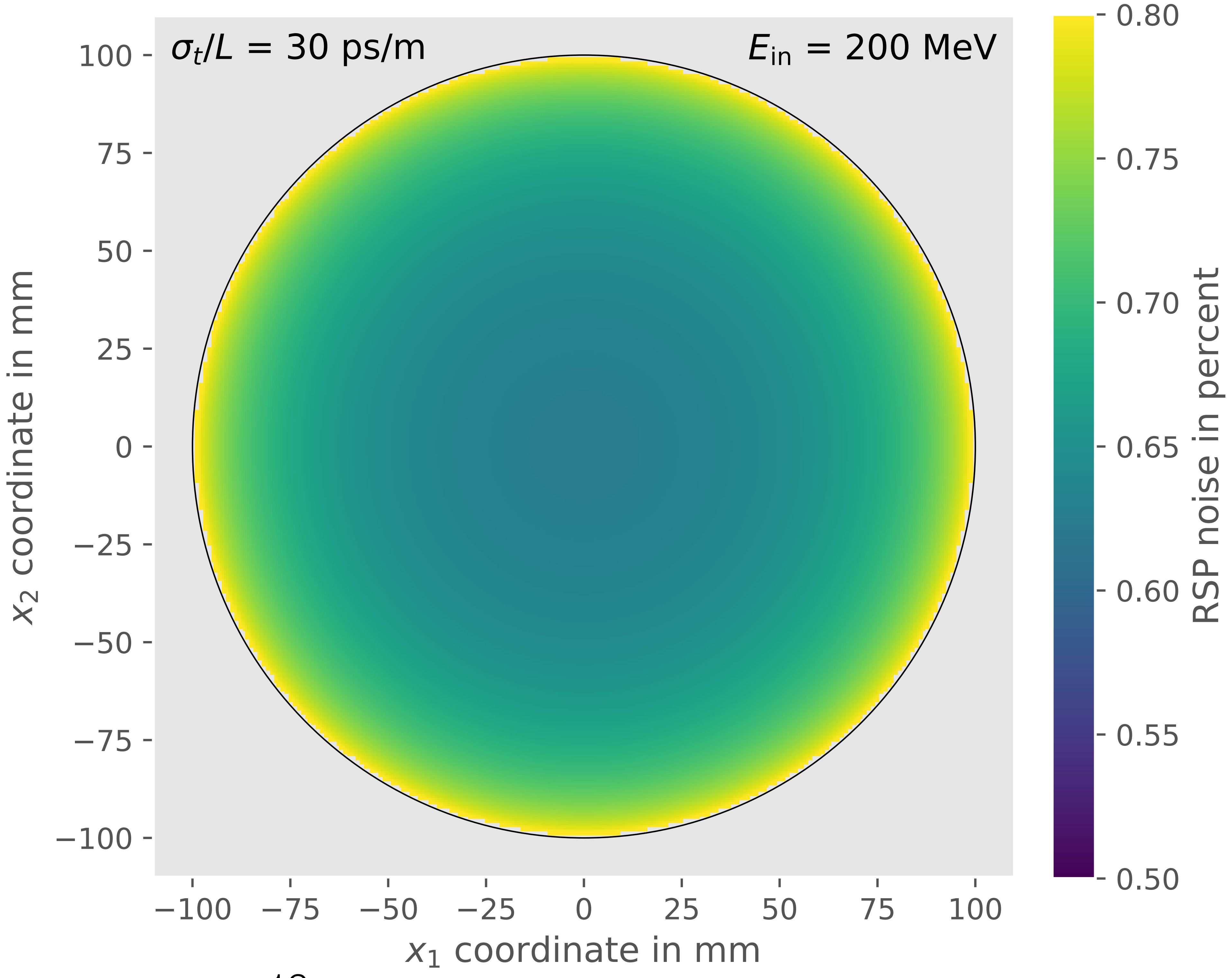
RSP uncertainty in a water cylinder

- Diameter: 20 cm
- Dose to center: 10 mGy (full acquisition)
- 1x1 mm^2 pixel size
- Incident beam energy: 200 MeV

calorimeter: 1% error



TOF velocity error: 30 ps/m



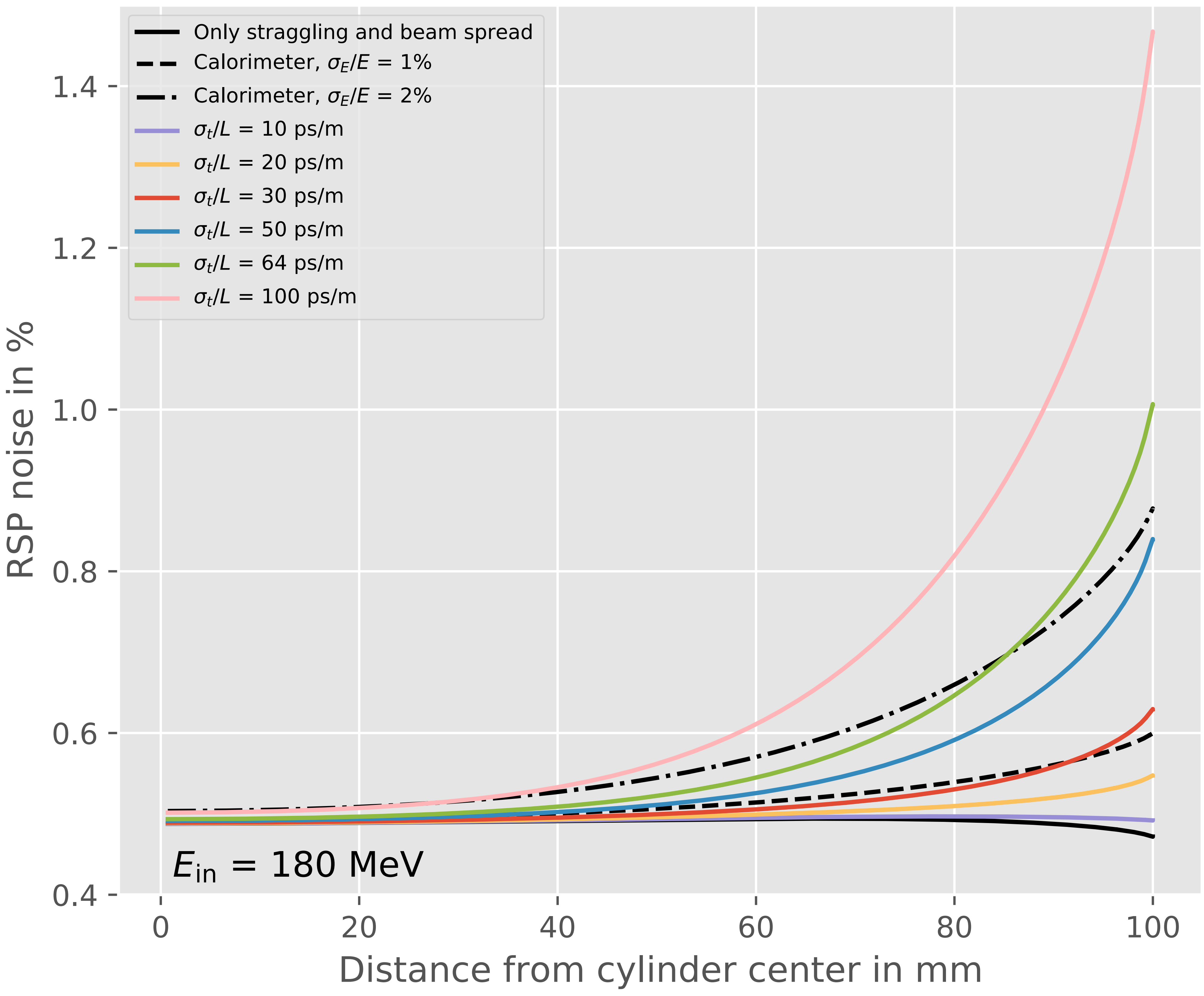
Images noisier towards the edges also with calorimeter-based system

Reason for 5-stage system in phase II pCT scanner (Bashkirov et al. 2016)

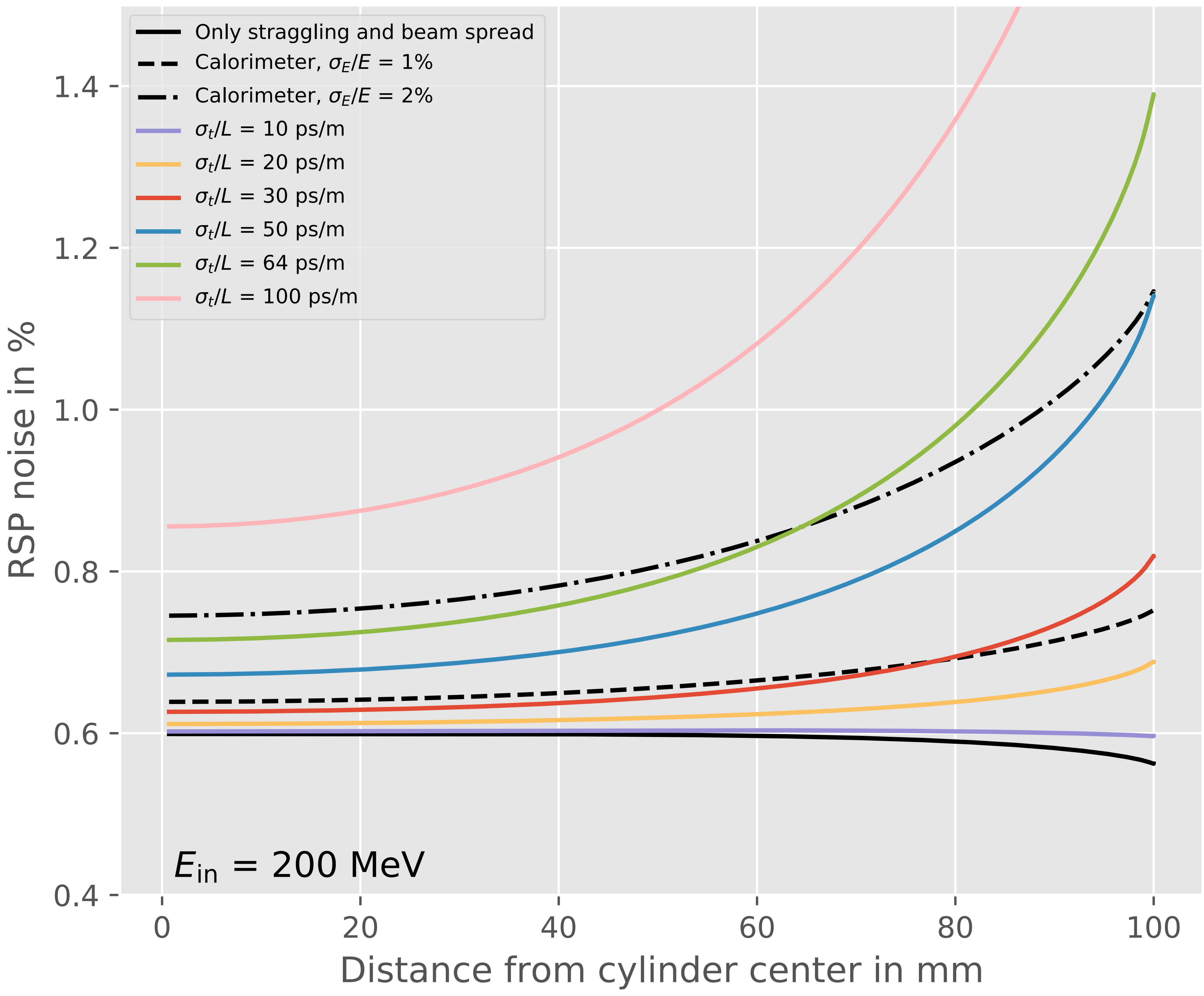
RSP uncertainty in a water cylinder

- Diameter: 20 cm
- Dose to center: 10 mGy (full acquisition)

Beam energy: 180 MeV

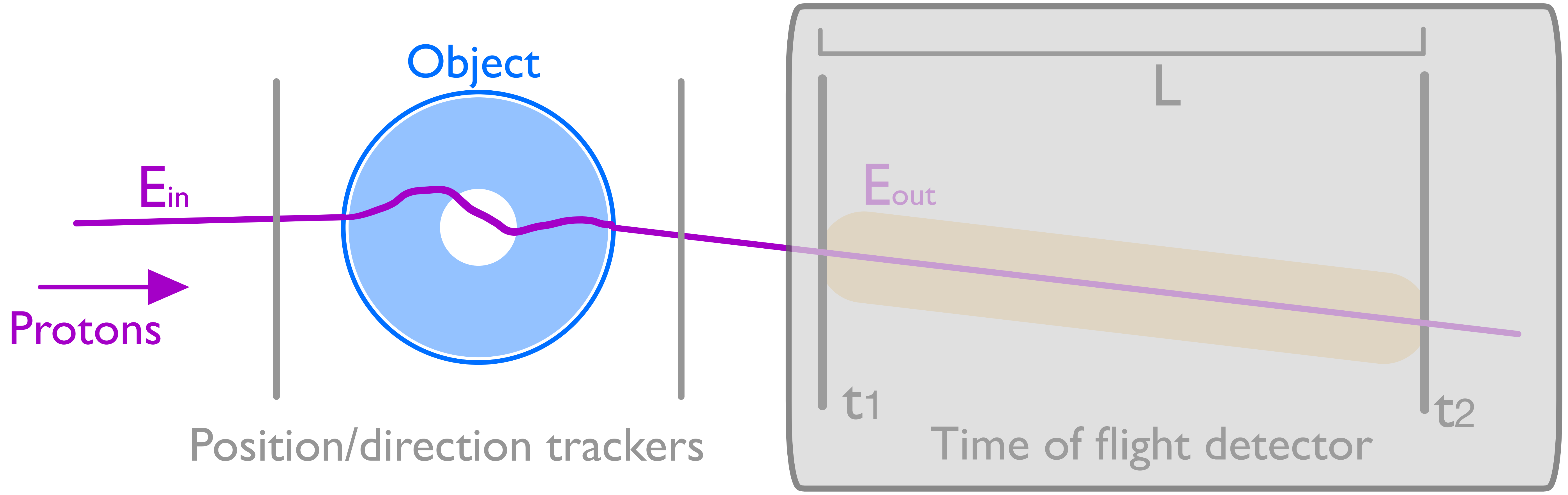


Beam energy: 200 MeV



Monte Carlo simulations

- Geant4/GATE simulation
- Dose to center: 10 mGy (full acquisition)
- Phantom: water cylinder with 20 cm diameter
- QGSP_BIC physics list and ideal selection of protons which have only undergone electromagnetic interactions
- Ideal position and direction scoring



- Simulated as ideal energy detector
- Energy uncertainty added in post-processing

100 independent simulations → **100 independent reconstructed images** → **Pixel-wise root-mean-square-error of RSP values** → **Direct estimate of RSP noise**

Noise due to multiple Coulomb scattering

- Multiple Coulomb scattering (MCS) deviates ions onto stochastic non-linear paths.
- Ions binned into the same pixel have traversed different phantom regions.
- This leads to WEPL variation if density gradients are present and near the object's edge [1,2].

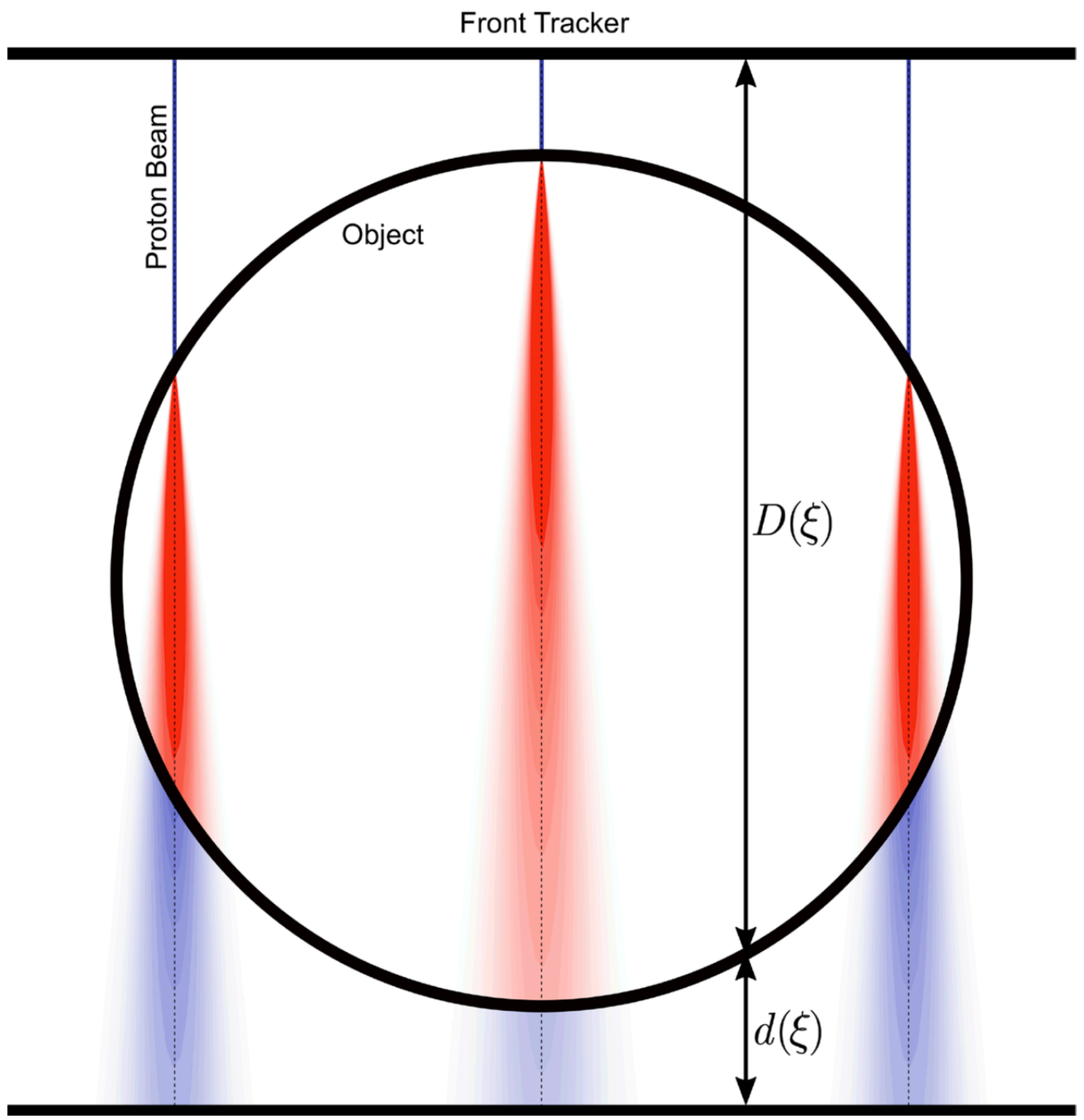


Image adapted from [1]

Estimated MCS contribution from Monte Carlo results:

contains all noise contributions

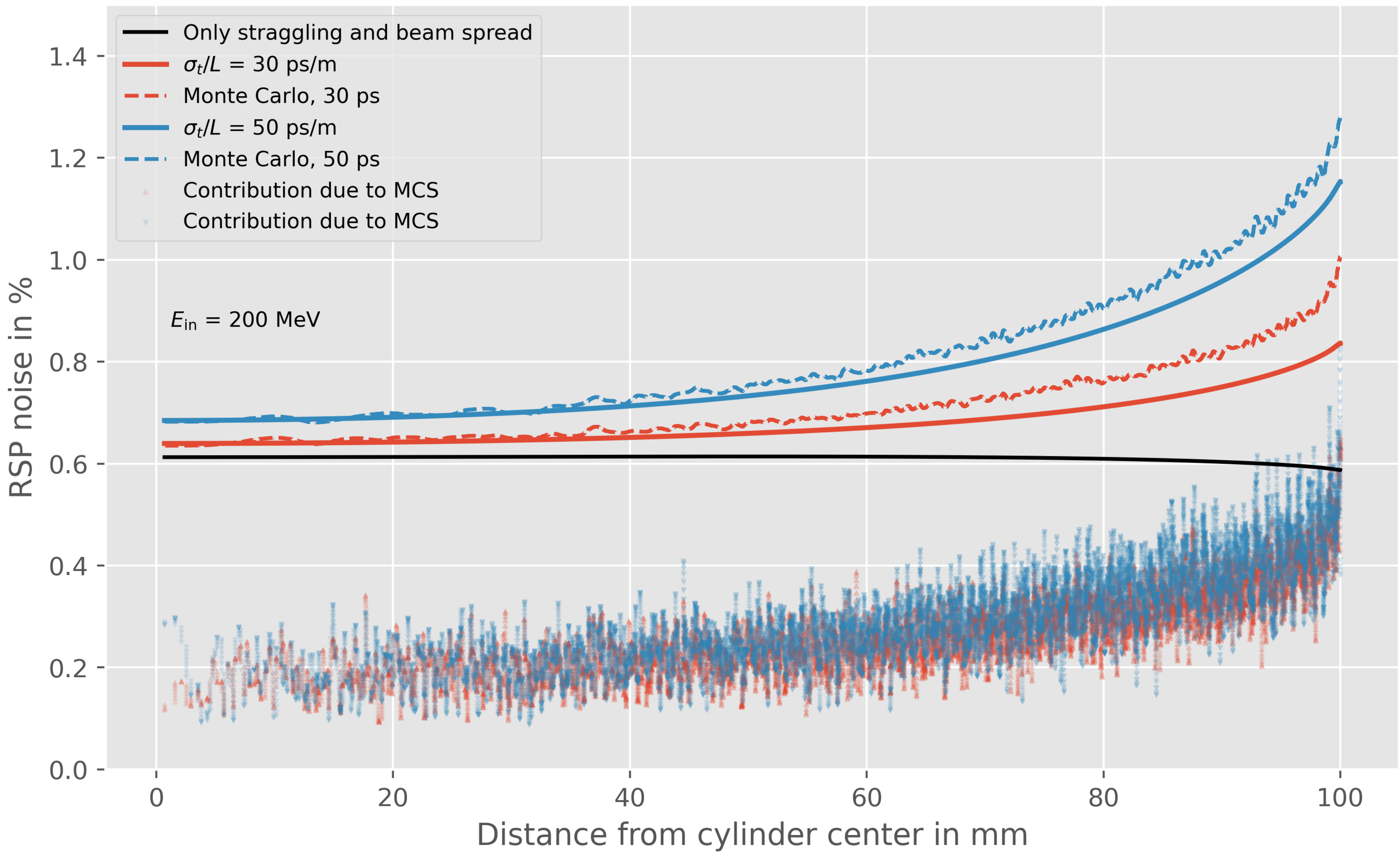
$$\mathrm{Var}_{\mathrm{RSP,MCS}} = \mathrm{Var}_{\mathrm{RSP,MC}} - \mathrm{Var}_{\mathrm{RSP,model}}$$

contains all noise contributions except for MCS

[1] Rädler, M. et al. (2018). Two-dimensional noise reconstruction in proton computed tomography using distance-driven filtered back-projection of simulated projections. Physics in Medicine & Biology, 63(21), 215009. https://doi.org/10.1088/1361-6560/aae5c9

[2] Dickmann, J., Wesp, P., Rädler, M., Rit, S., Pankuch, M., Johnson, R. P., … Dedes, G. (2019). Prediction of image noise contributions in proton computed tomography and comparison to measurements. Physics in Medicine & Biology, 64(14), 145016. https://doi.org/10.1088/1361-6560/ab2474

Monte Carlo results



Proton vs helium

Observations:

Helium mass = 4 x proton mass

Helium stopping power = 4 x proton stopping power

At equal residual range:
helium beam energy = 4 x proton beam energy

All energy and mass terms scale by factor of 4!

$$\sigma^2_{\mathrm{WEPL,He}} = \frac{1}{S^2_{\mathrm{w,p}}\Phi\Delta\xi^2}\left(\frac{1}{4}\sigma^2_{E_{\mathrm{out}},\mathrm{strag,p}} + \frac{\sigma^2_{t,\mathrm{He}}}{\sigma^2_{t,\mathrm{p}}}\sigma^2_{E_{\mathrm{out}},\mathrm{TOF,p}} + (\delta E_{\mathrm{beam,He}}E_{\mathrm{in,p}})^2\right)$$

Ratio of measurement errors:

$$\frac{\sigma_{t,\mathrm{He}}}{\sigma_{t,\mathrm{p}}} \approx \frac{1}{4}$$

because detector response scales with stopping power

Proton vs helium: at equal dose

$$\sigma^2_{\mathrm{WEPL,He}} \approx \frac{1}{S^2_{\mathrm{w,p}}\Phi_{\mathrm{helium}}\Delta\xi^2}\left(\frac{1}{4}\sigma^2_{E_{\mathrm{out}},\mathrm{strag,p}} + \frac{1}{4}\sigma^2_{E_{\mathrm{out}},\mathrm{TOF,p}} + (\delta E_{\mathrm{beam,He}}E_{\mathrm{in,p}})^2\right)$$

Dose scales with stopping power:

$D \propto S$ **and** $S_{\mathrm{helium}} \approx 4S_{\mathrm{proton}}$

Therefore, at equal dose:

$$\frac{1}{\Phi_{\mathrm{helium}}} \approx 4\frac{1}{\Phi_{\mathrm{proton}}}$$

$$\sigma^2_{\mathrm{WEPL,He}} = \frac{1}{S^2_{\mathrm{w,p}}\Phi_{\mathrm{proton}}\Delta\xi^2}\left(\sigma^2_{E_{\mathrm{out}},\mathrm{strag,p}} + \sigma^2_{E_{\mathrm{out}},\mathrm{TOF,p}} + 4(\delta E_{\mathrm{beam,He}}E_{\mathrm{in,p}})^2\right)$$

Protons and helium ions expected to yield similar noise.

Conclusion

- Time-of-flight is an alternative method for energy-loss measurement in ion CT
- RSP resolution better than 1% with velocity errors <50 ps/m
- At 30-50 ps/m velocity error: image noise is comparable with calorimeter-based system with 1-2% error
- Noise can be improved by optimizing incident beam energy as a function of expected water equivalent path length, e.g. via optimization similar to Dickmann et al. 2019 (see also talk by George)
- Image noise expected to be similar with protons and helium ions.
- Interesting novel sensor technology from field of particle physics, e.g. LGAD (see talk by Stefanie)

Thanks

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MAIN MENU

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WELCOME TO THE THIRD ION IMAGING WORKSHOP 2022

The ion imaging workshop 2022 will take place in Munich, Germany. It is the third edition after the workshops in 2018 and 2019.

Date: October 13-14, 2022

Venue: LMU Munich, Germany

Important dates:

Registration is now open.

Registration fee is 180 Euro including lunches, coffee breaks, and the social dinner.

Registration deadline: September 1, 2022.

Abstract submission deadline: ~~July 15, 2022~~, extended to July 27, 2022.