Time-of-flight proton CT

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Physics in Medicine & Biology

ACCEPTED MANUSCRIPT

Relative stopping power resolution in time-of-flight proton CT

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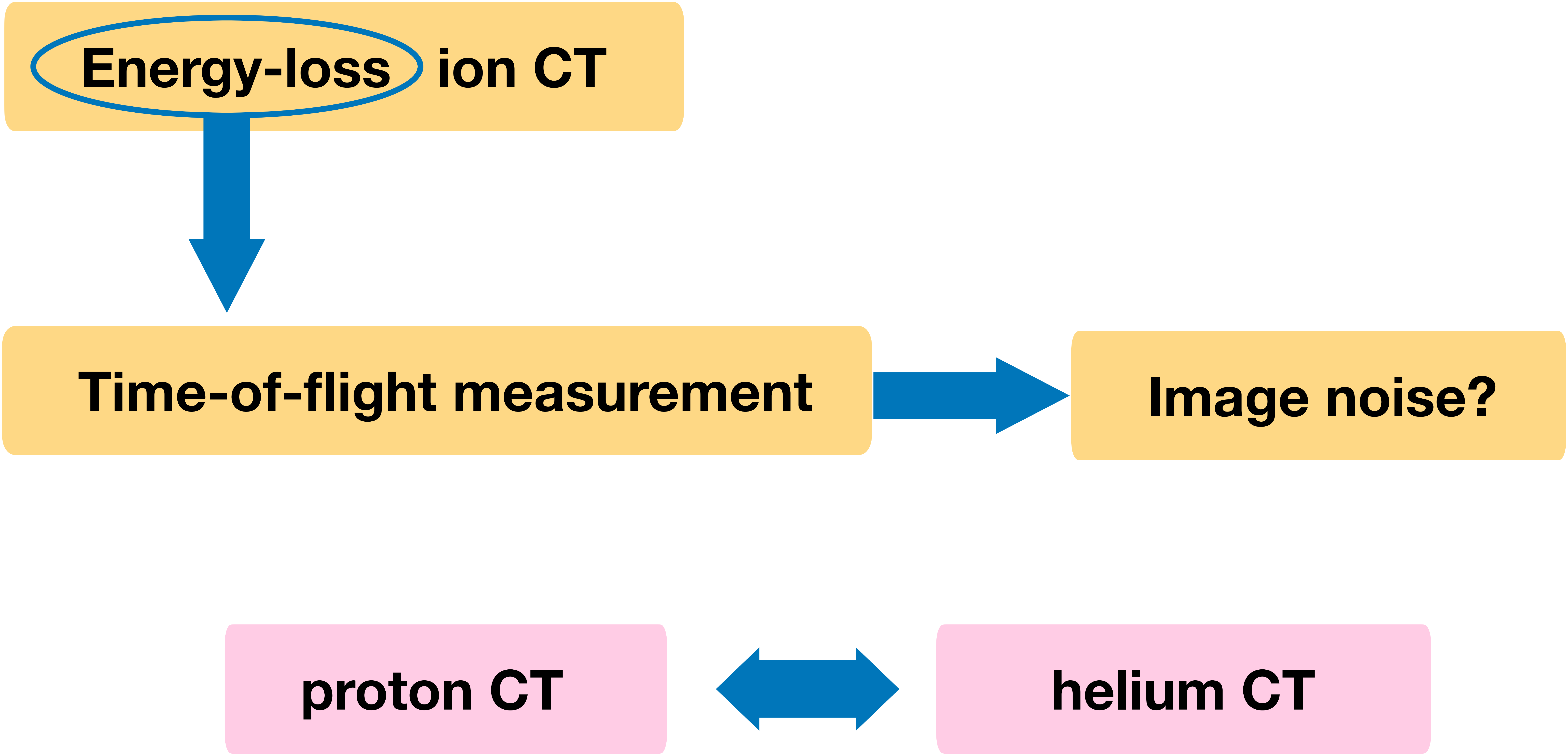
What is an Accepted Manuscript?

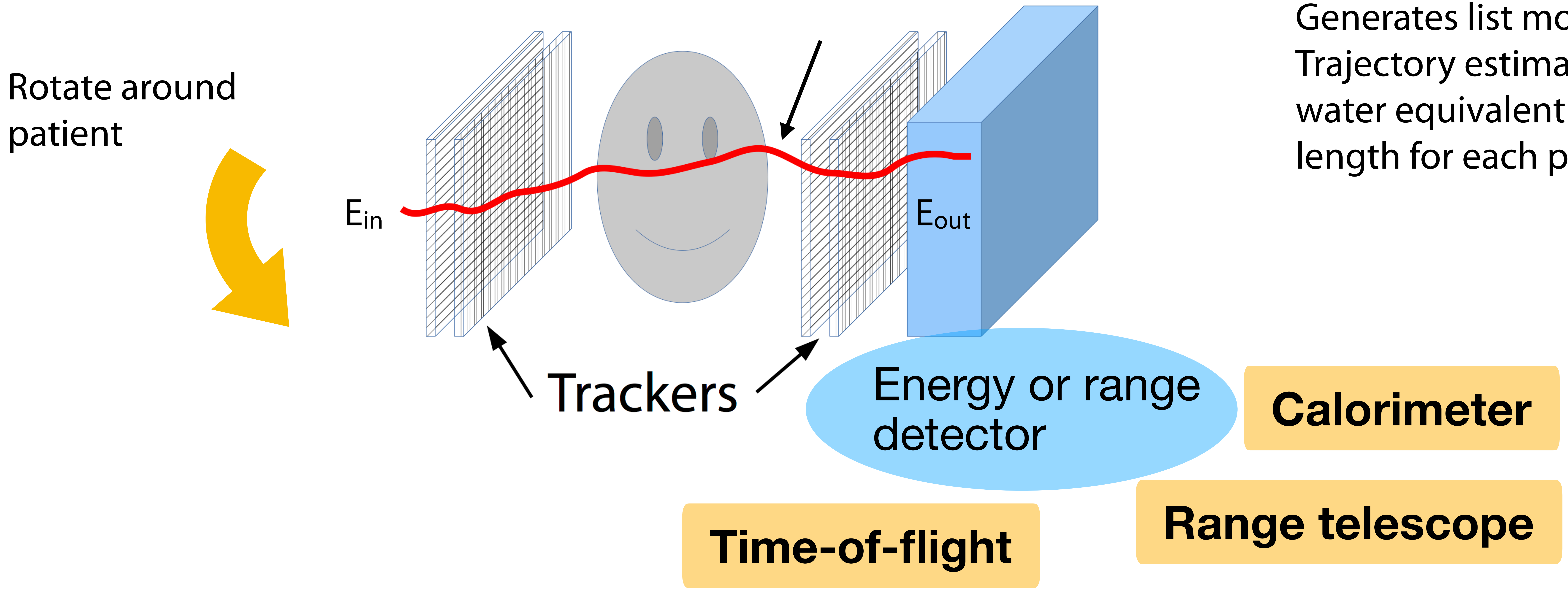
Accepted Manuscript PDF

DOI: 10.1088/1361-6560/ac7191

I will speak about …



Typical list-mode ion CT set-up



Generates list mode data: Trajectory estimate and water equivalent path length for each proton.

Comprehensive review:

Johnson, R. P. (2018). Review of medical radiography and tomography with proton beams. Reports on Progress in Physics, 81(1), 016701. https://doi.org/10.1088/1361-6633/aa8b1d

Reconstruction problem in ion CT

Estimated ion path across object

Measured by range telescope

or

Measured by calorimeter or time-of-flight detector

$$\int_{\Gamma} \mathrm{RSP}(x)\mathrm{d}x = \mathrm{WEPL}(E_{\mathrm{out}}) = \int_{E_{\mathrm{in}}}^{E_{\mathrm{out}}} 1/S_w(E)\mathrm{d}E$$

Relative stopping power: material property

Water equivalent path length

Stopping power in water

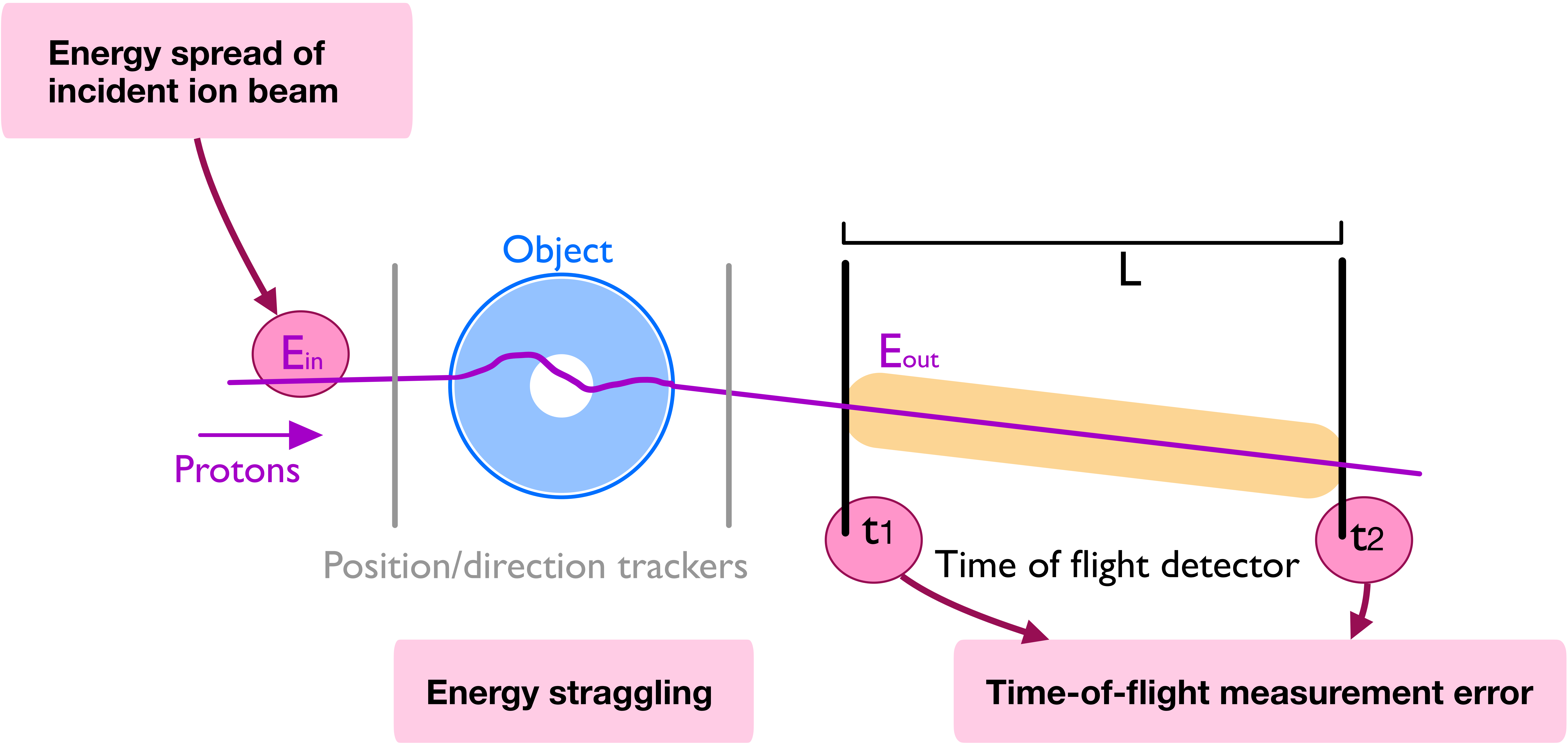
Question to be answered:

What is the impact of energy uncertainty on the estimated RSP map in terms of noise?

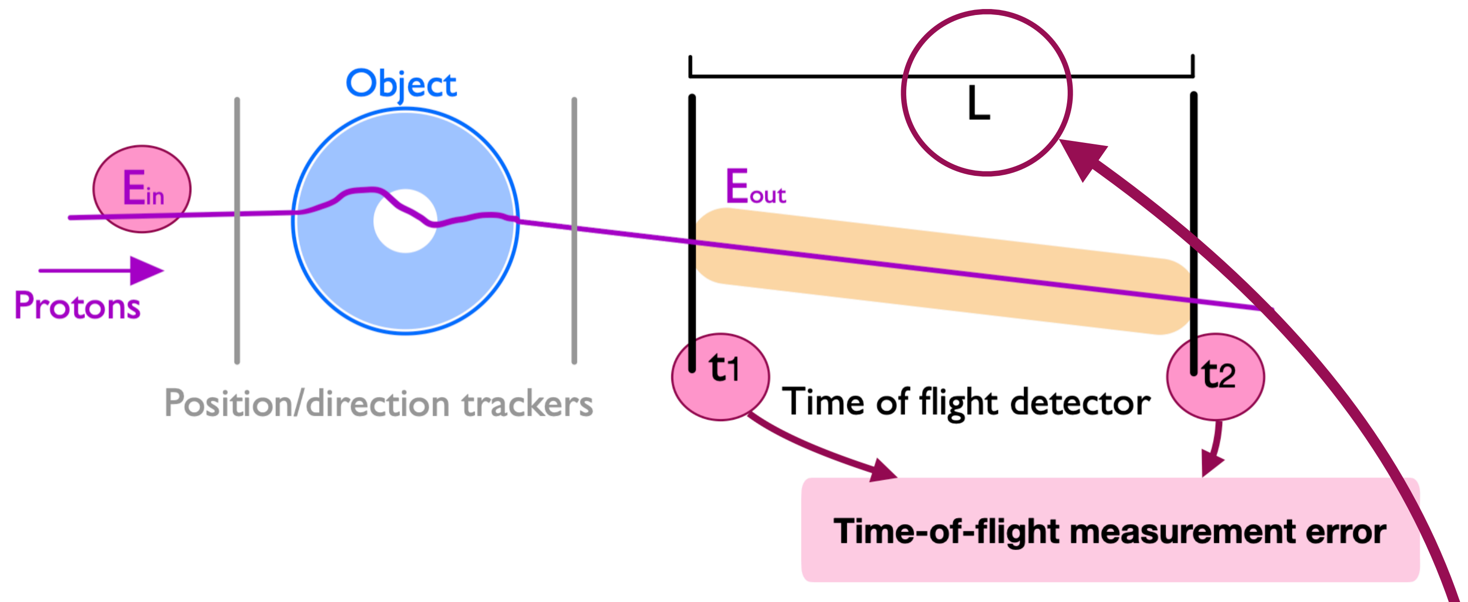
$$\int_{\Gamma} \mathrm{RSP}(x)\mathrm{d}x = \mathrm{WEPL}(E_{\mathrm{out}}) = \int_{E_{\mathrm{in}}}^{E_{\mathrm{out}}} 1/S_w(E)\mathrm{d}E$$

Error propagation

Sources of energy error/uncertainty



Time-of-flight measurement error



Relativistic energy - velocity relation:

$$E_{\text{out}} = \frac{m_p c^2}{\sqrt{1-(v/c)^2}} - m_p c^2 \quad \text{with} \quad v = \frac{L}{t_2 - t_1},$$

time-of-flight

First order error propagation:

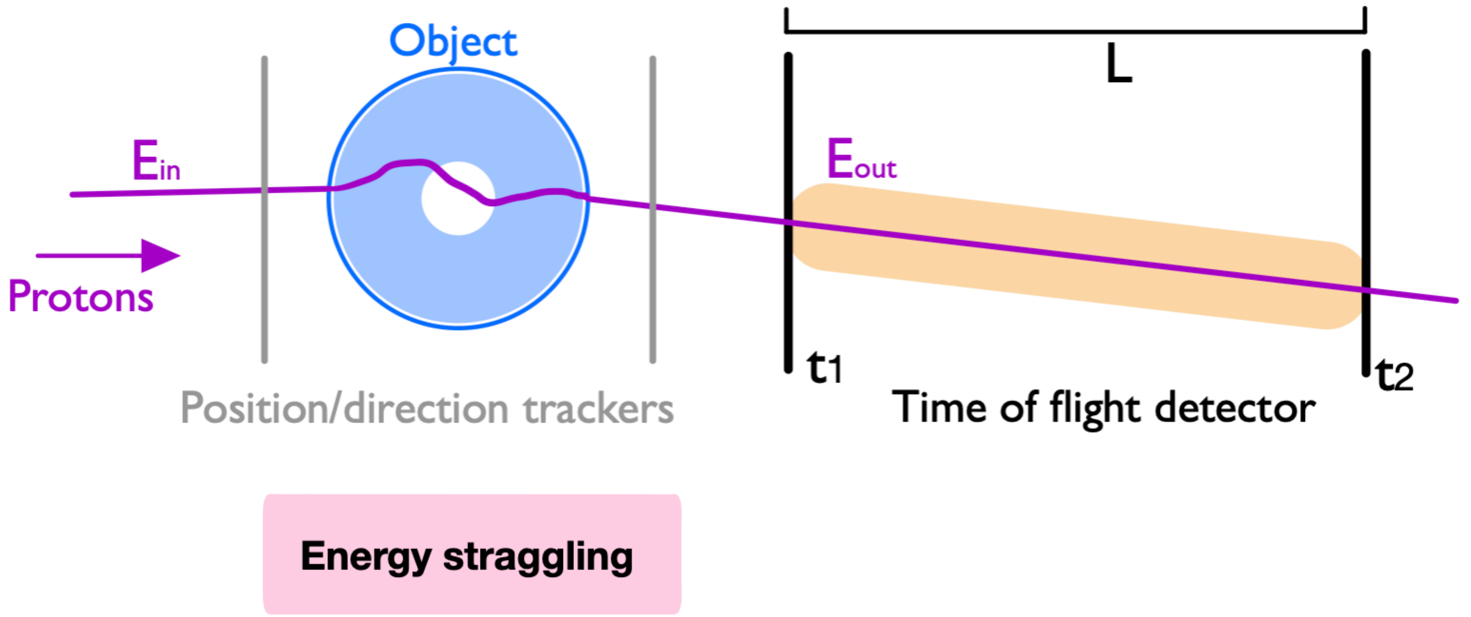
$$\sigma^2_{E_{\text{out}},\text{TOF}}(E_{\text{out}}) = \left|\frac{\mathrm{d}E}{\mathrm{d}t_1}\right|^2 \sigma^2_{t_1} + \left|\frac{\mathrm{d}E}{\mathrm{d}t_2}\right|^2 \sigma^2_{t_2} = \frac{1}{m_p^4 c^6}(E^2_{\text{out}} + 2m_p c^2 E_{\text{out}})^3 \frac{\sigma_t^2}{L^2}$$

energy error (variance)

velocity error (variance)

Note: $\sigma^2_{E_{\text{out}},\text{TOF}} \propto E^3_{\text{out}}$

Energy straggling



- **Variation of energy loss within ensemble of ions due to stochastic nature of electromagnetic interactions.**
- **Approximately Gaussian energy distribution.**
- **Variance can be calculated analytically (to first order) [1].**

energy error (variance)

$$\sigma^2_{E_{\mathrm{out}},\mathrm{straggling}}(E_{\mathrm{out}}) = \chi_1^2(E_{\mathrm{out}}) \int_{E_{\mathrm{out}}}^{E_{\mathrm{in}}} \frac{\chi_2(E)}{\chi_1^3(E)} \mathrm{d}E$$

Solve via numerical integration.

$$\chi_1(E) = K \frac{1}{\beta^2} \left[\ln \left(\frac{2 m_e c^2 \beta^2}{I(1-\beta^2)} \right) - \beta^2 \right] \quad \text{with} \quad \beta = \frac{v}{c} = \left[1 - \left(\frac{m_p c^2}{m_p c^2 + E} \right)^2 \right]^{1/2}$$

$$\chi_2(E) = K m_e c^2 \frac{1 - \beta^2/2}{1 - \beta^2}$$

I: ionisation potential (approx. as water, 75 eV)
m_p: proton mass
m_e: electron mass
K: a constant

[1] Payne, M. G. (1969). Energy Straggling of Heavy Charged Particles in Thick Absorbers. Physical Review, 185(2), 611–623. DOI: 10.1103/PhysRev.185.611

Comparison with calorimeter

time-of-flight

$$\sigma^2_{E_{\rm out}}(E_{\rm out}) = \sigma^2_{E_{\rm out},\rm straggling}(E_{\rm out}) + \sigma^2_{E_{\rm out},\rm TOF}(E_{\rm out})$$

Compare with calorimeter-based ion CT system [1]:

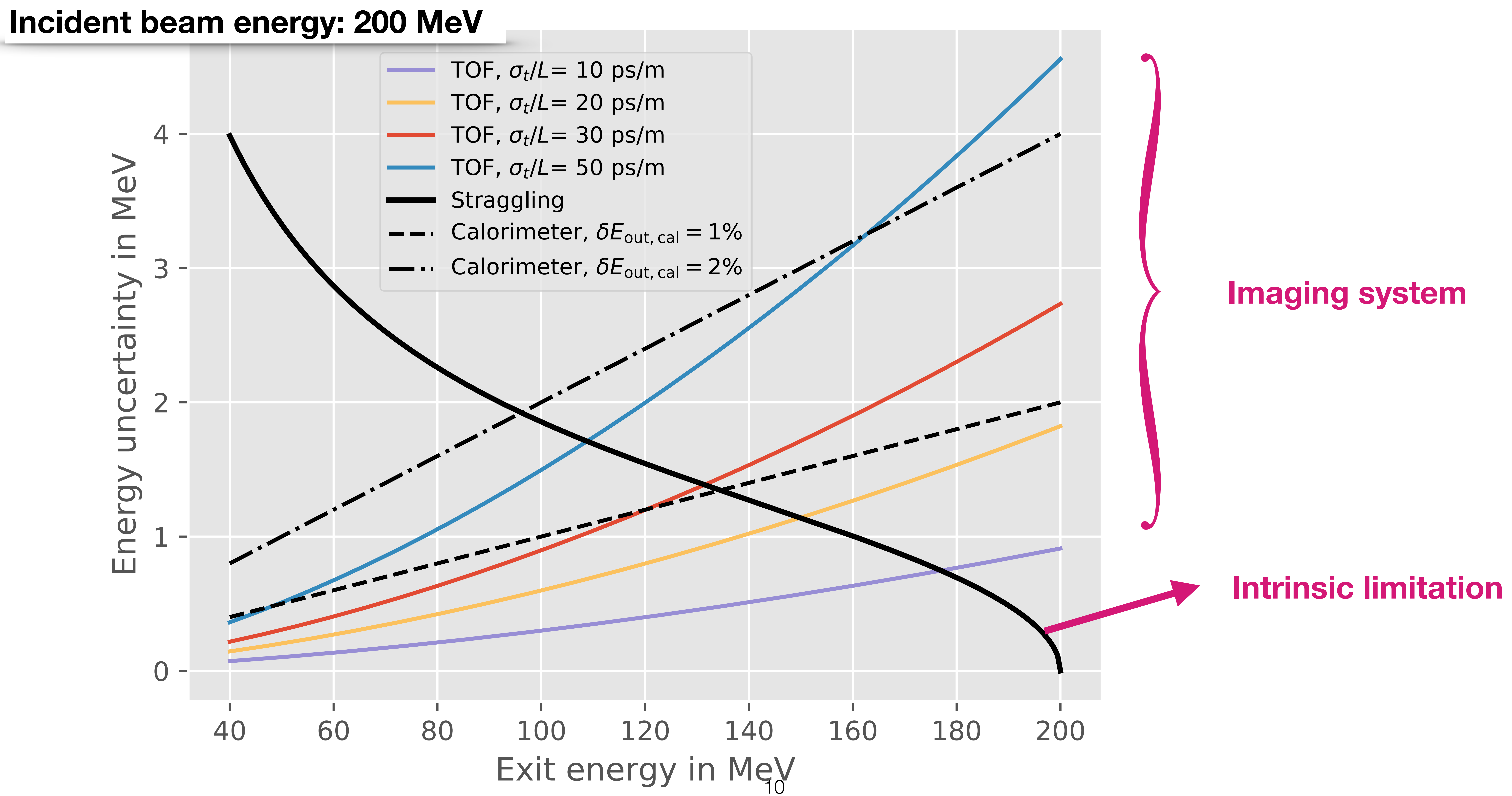
$$\sigma^2_{E_{\rm out},\rm cal}(E_{\rm out}) = \sigma^2_{E_{\rm out},\rm straggling}(E_{\rm out}) + \delta^2 E_{\rm out,cal} E^2_{\rm out}$$

calorimeter

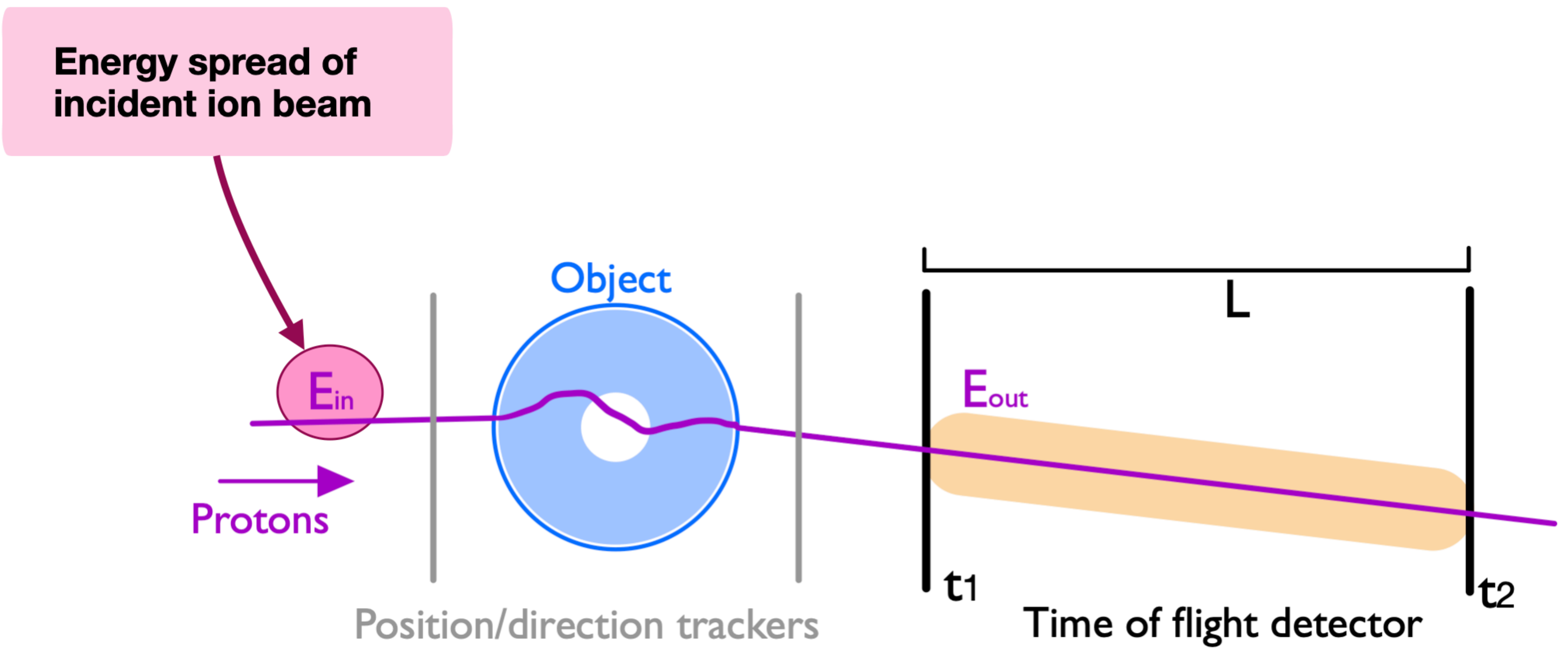
$$\delta E_{\rm out,cal} \approx 1-2\,\%$$

[1] Bashkirov, V. A. et al. (2016). Novel scintillation detector design and performance for proton radiography and computed tomography. Medical Physics, 43(2), 664–674. https://doi.org/10.1118/1.4939255

Energy uncertainty



Energy spread of incident beam



- Depends on accelerator and beam delivery system.
- E.g. synchrotron vs. cyclotron
- We assumed 0.5% of beam energy [1].

$$\sigma^2_{\text{beam}}(E_{\text{in}}) = \delta^2 E_{\text{beam}} E^2_{\text{in}} \quad \text{with} \quad \delta E_{\text{beam}} = 0.5\,\%$$

[1] Schippers, J. M. (2018). Beam Transport Systems for Particle Therapy. In R. Bailey (Ed.), Proceedings of the CAS-CERN Accelerator School: Accelerators for Medical Applications,. Vösendorf, Austria: CERN. https://doi.org/10.23730/CYRSP-2017-001.241

WEPL uncertainty

$$\sigma^2_{\Delta E}(E_{\text{out}}) = \sigma^2_{E_{\text{out}},\text{straggling}}(E_{\text{out}}) + (\delta E_{\text{beam}} E_{\text{in}})^2 + \sigma^2_{E_{\text{out}},\text{TOF}}(E_{\text{out}})$$

+ multiple Coulomb scattering

First order error propagation:

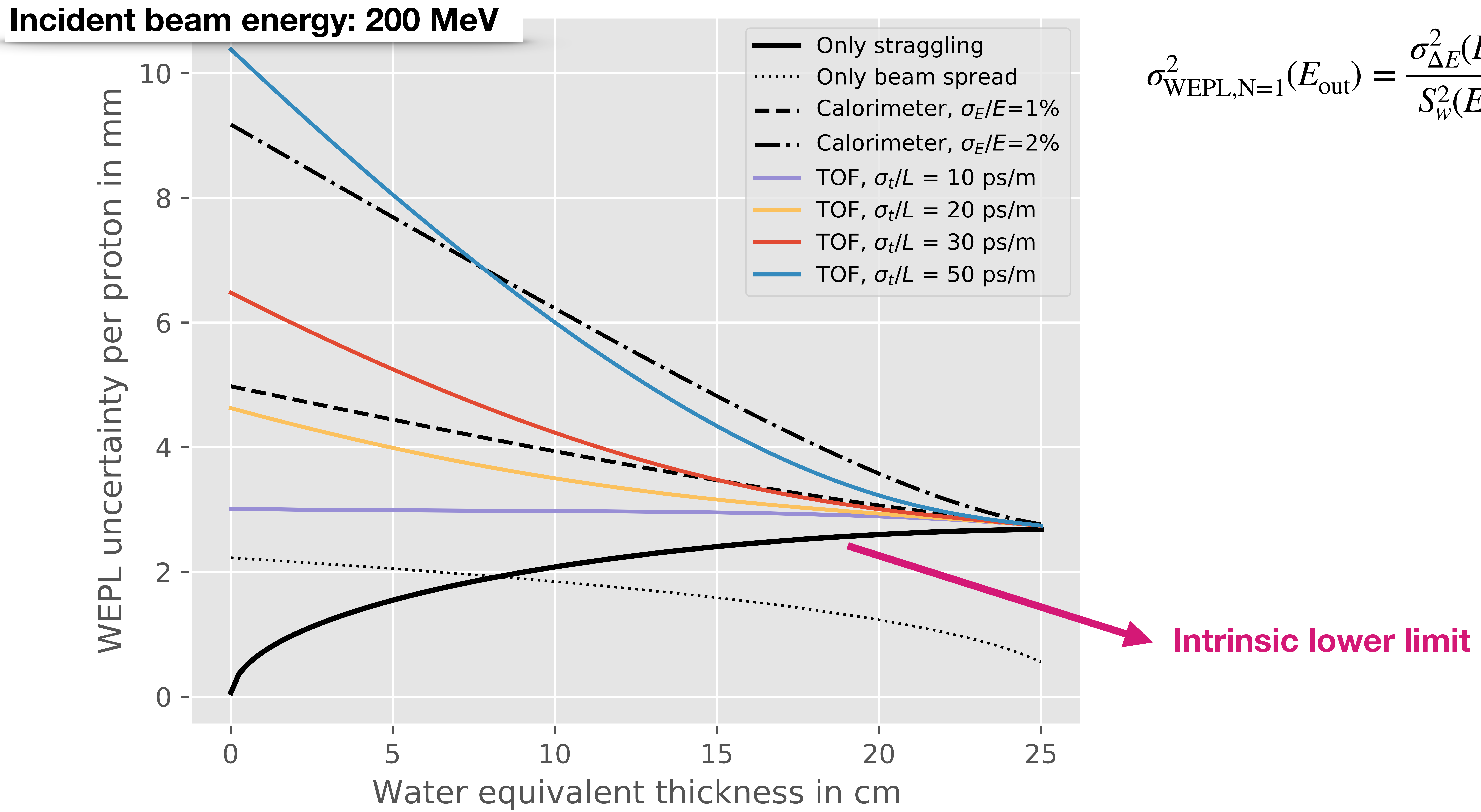
$$\text{WEPL}(E_{\text{out}}) = \int_{E_{\text{in}}}^{E_{\text{out}}} 1/S_w(E)\,\mathrm{d}E \quad \Rightarrow \quad \sigma^2_{\text{WEPL}}(E_{\text{out}}) = \frac{\sigma^2_{\Delta E}(E_{\text{out}})}{S^2_w(E_{\text{out}})N} = \frac{\sigma^2_{\Delta E}(E_{\text{out}})}{S^2_w(E_{\text{out}})\,\Phi\,\Delta\xi^2}$$

N: number of ions

Φ: particle fluence (dose)

$\Delta\xi^2$: pixel size, e.g. 1 mm²

WEPL uncertainty



$$\sigma^2_{\mathrm{WEPL,N=1}}(E_{\mathrm{out}}) = \frac{\sigma^2_{\Delta E}(E_{\mathrm{out}})}{S^2_w(E_{\mathrm{out}})}$$

RSP uncertainty via noise reconstruction

Propagate noise from WEPL to RSP: $\mathrm{WEPL}(E_{\mathrm{out}}) = \int_{\Gamma} \mathrm{RSP}(x)\mathrm{d}x$

projection images containing WEPL variance → **noise reconstruction [1,2]** → **reconstructed images containing RSP variance**

Assumptions:

- Ion CT images are reconstructed via filtered backprojection …
- … in fan beam geometry
- Linear interpolation between pixels
- Filtered with an apodized ramp filter

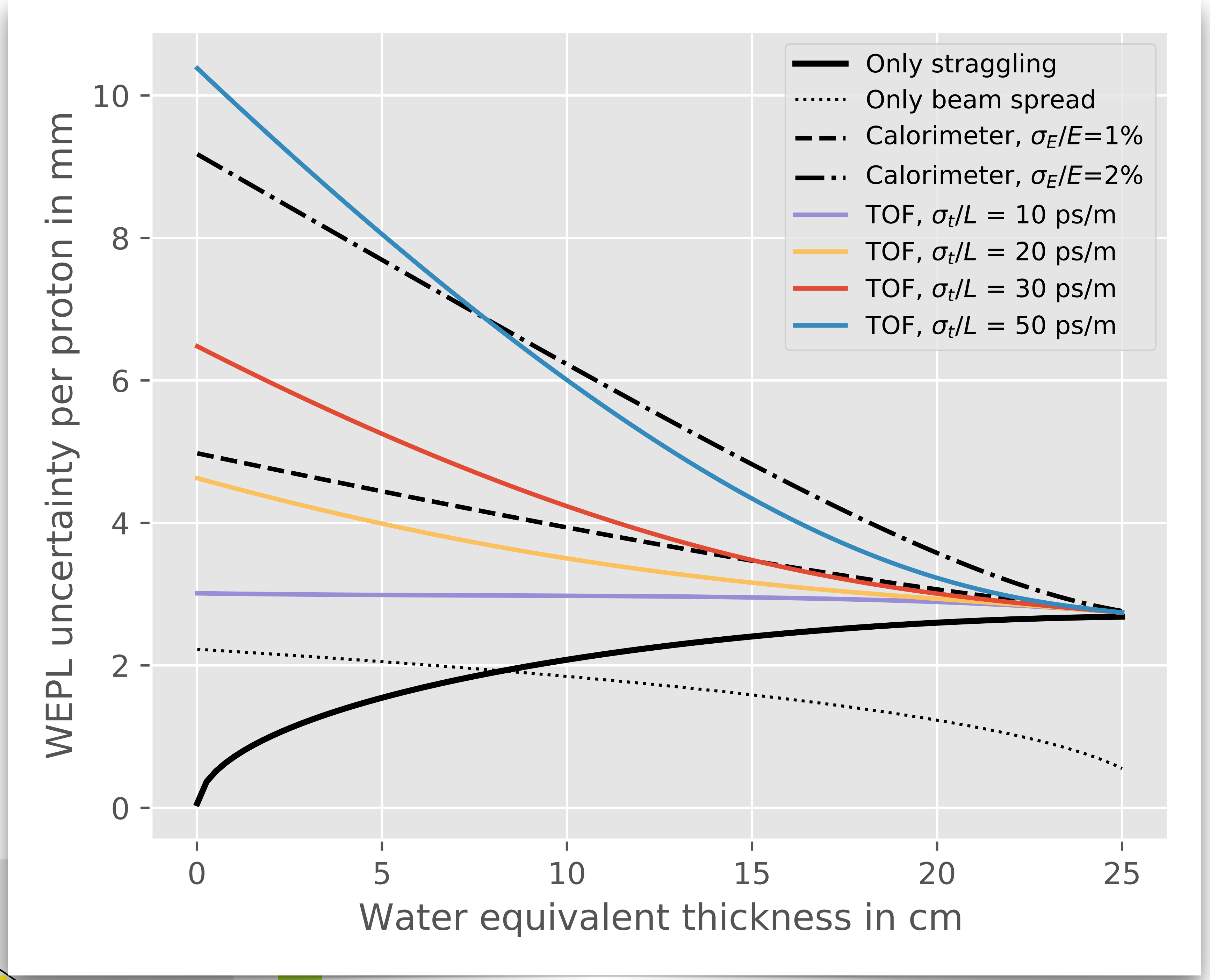
See Stefanie's talk yesterday

[1] Wunderlich, A., & Noo, F. (2008). Image covariance and lesion detectability in direct fan-beam x-ray computed tomography. Physics in Medicine and Biology, 53(10), 2471–2493. https://doi.org/10.1088/0031-9155/53/10/002

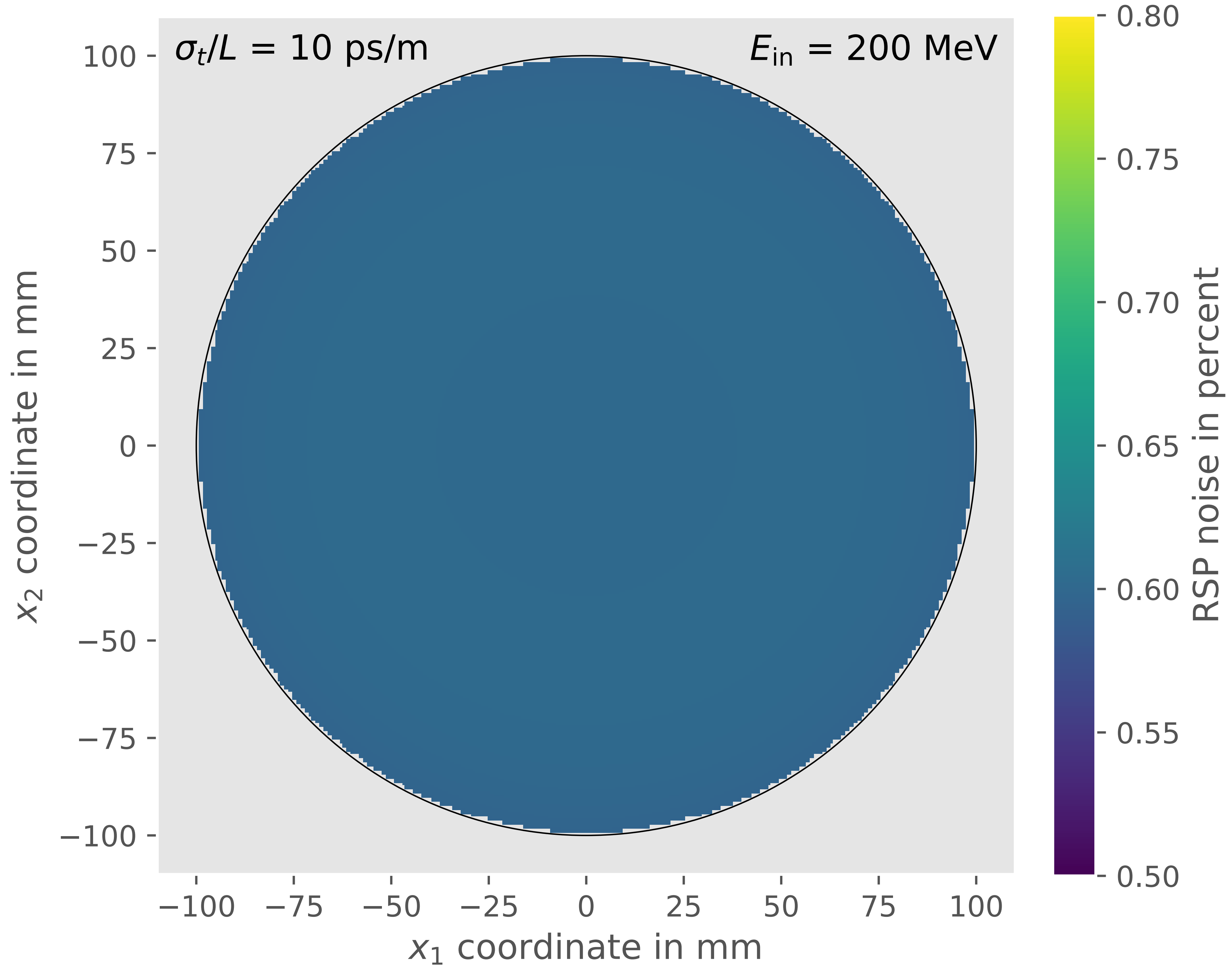
[2] Rädler, M. et al. (2018). Two-dimensional noise reconstruction in proton computed tomography using distance-driven filtered back-projection of simulated projections. Physics in Medicine & Biology, 63(21), 215009. https://doi.org/10.1088/1361-6560/aae5c9

RSP uncertainty in a water cylinder

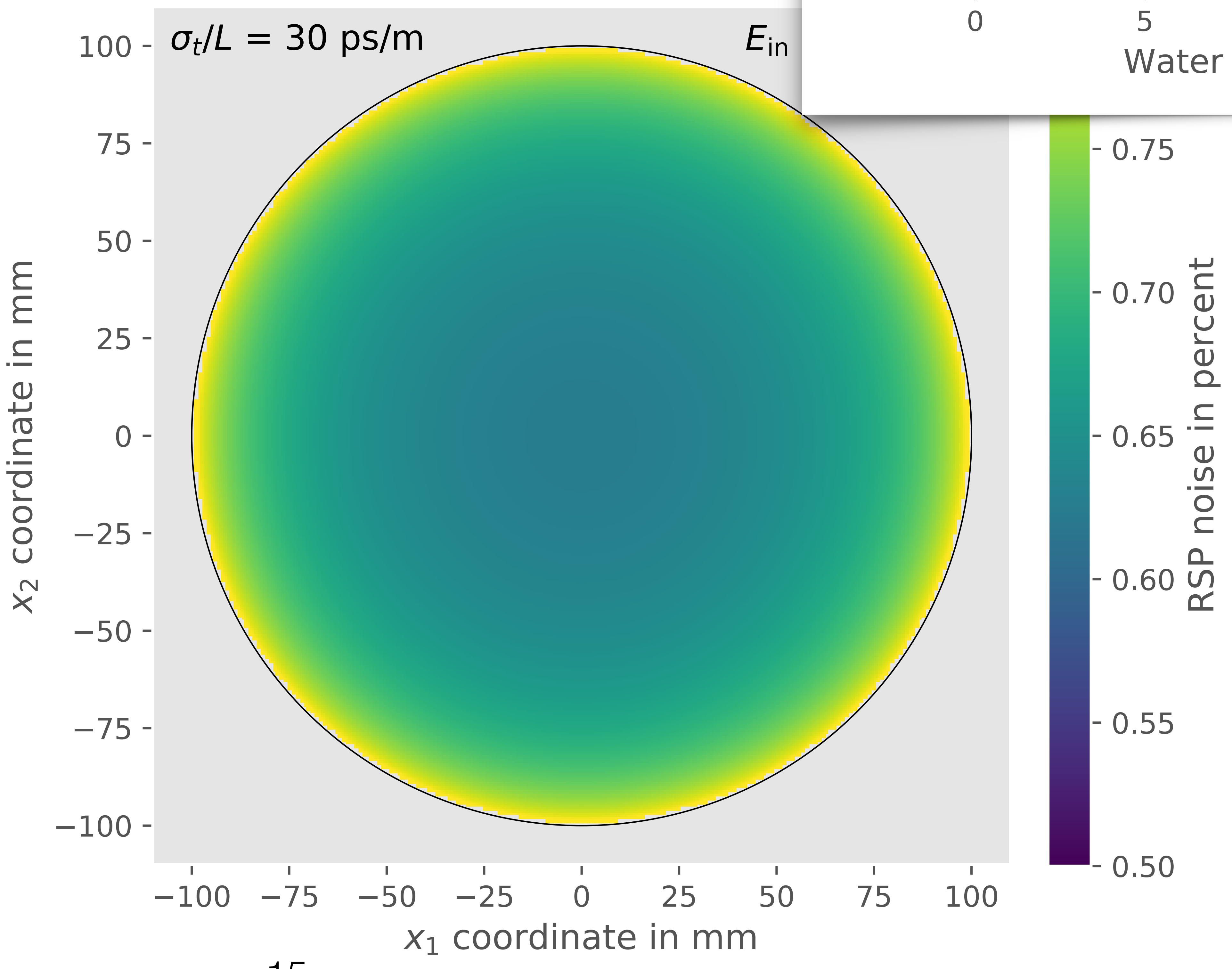
- Diameter: 20 cm
- Dose to center: 10 mGy (full acquisition)
- 1x1 mm^2 pixel size
- Incident beam energy: 200 MeV



velocity error: 10 ps/m



velocity error: 30 ps/m

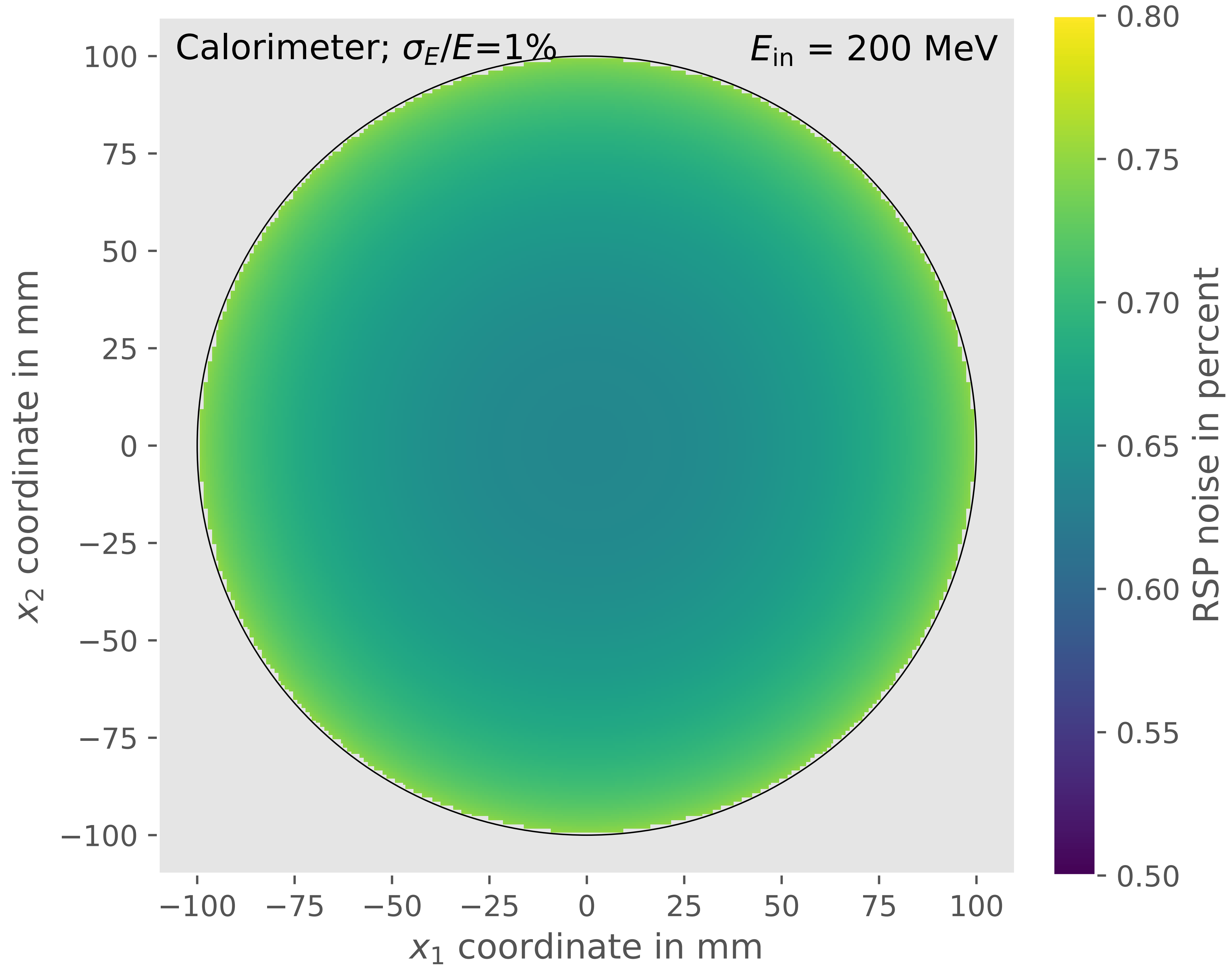


Images noisier towards the edges

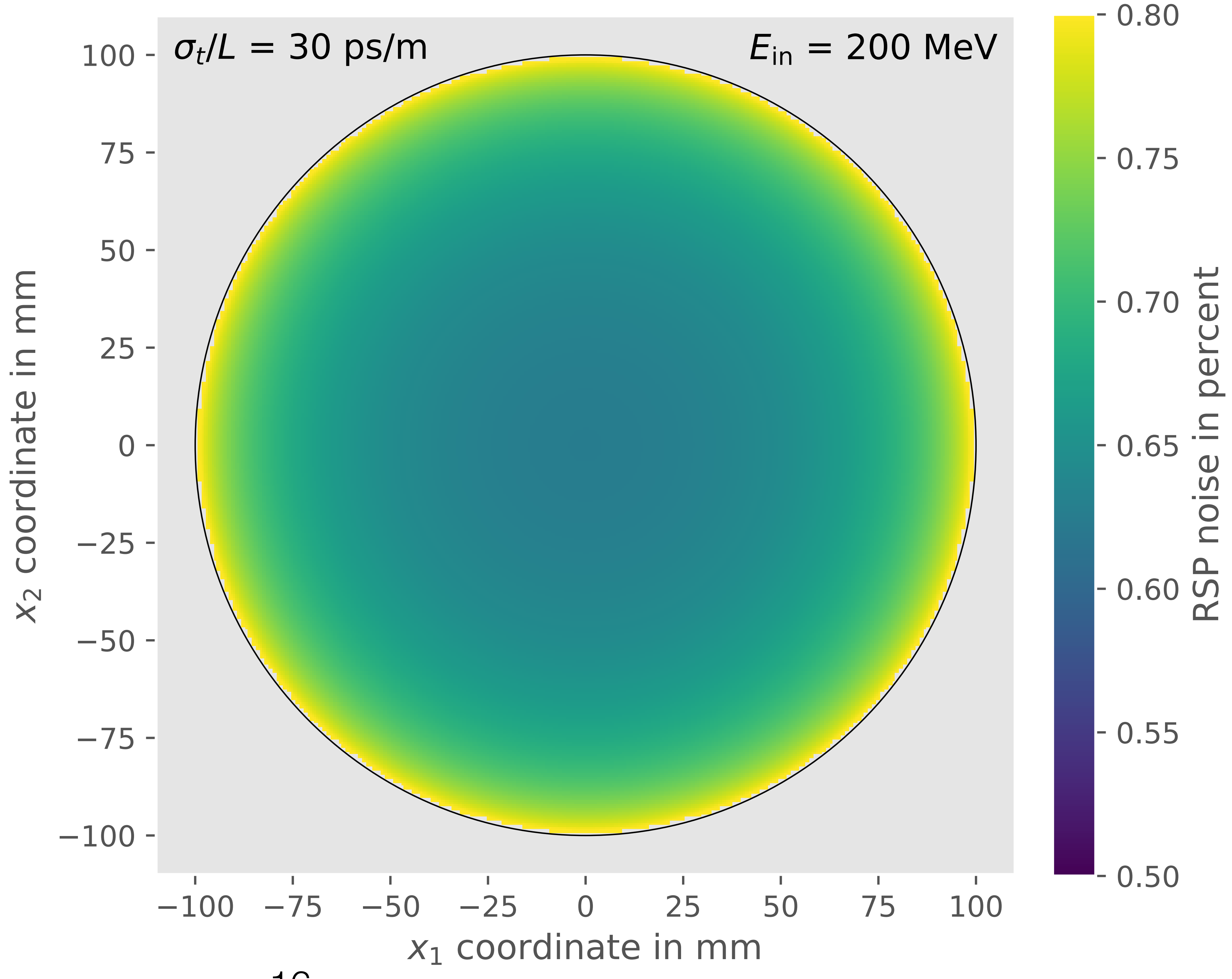
RSP uncertainty in a water cylinder

- Diameter: 20 cm
- Dose to center: 10 mGy (full acquisition)
- 1x1 mm^2 pixel size
- Incident beam energy: 200 MeV

calorimeter: 1% error



TOF velocity error: 30 ps/m



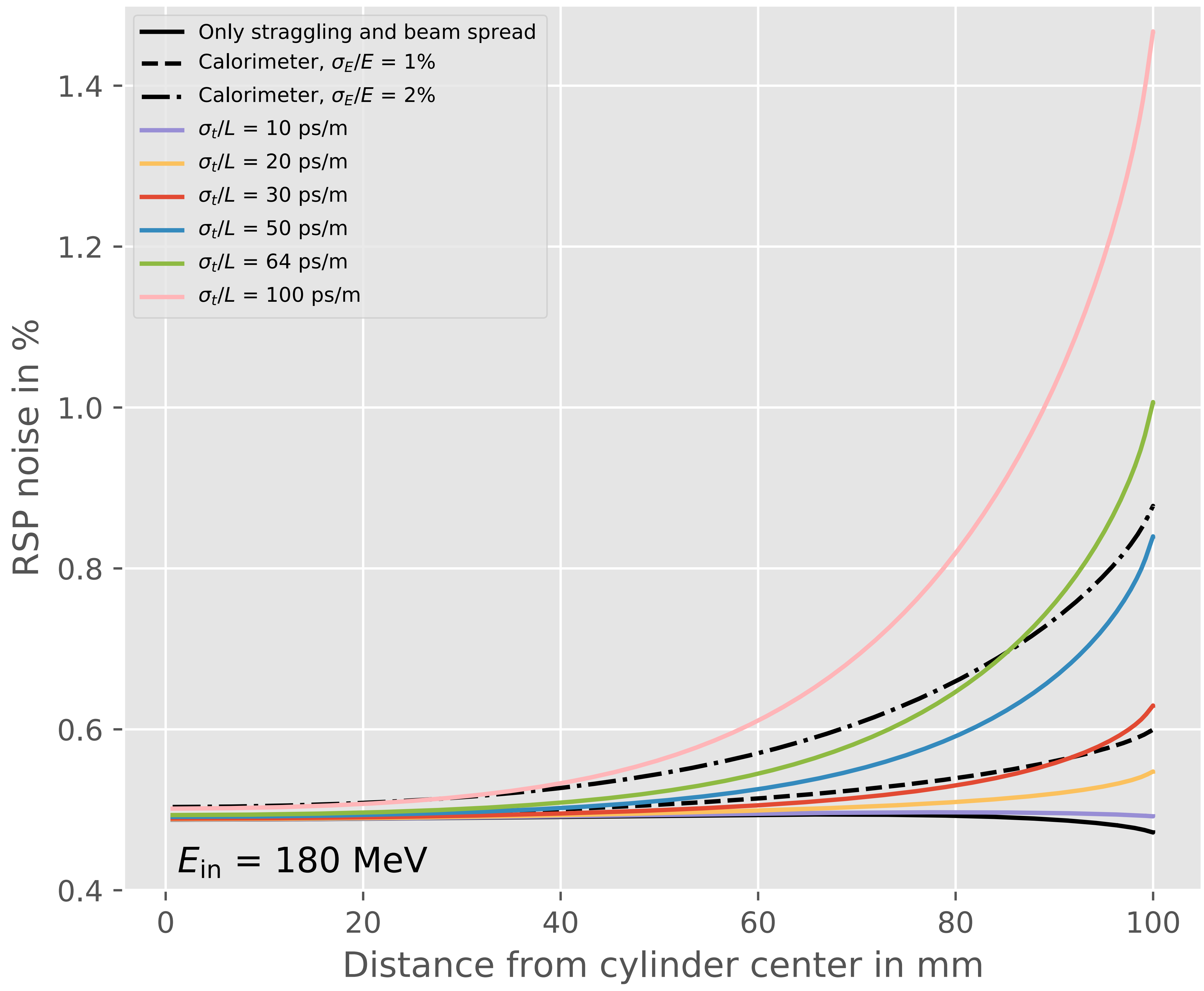
Images noisier towards the edges also with calorimeter-based system

Reason for 5-stage system in phase II pCT scanner (Bashkirov et al. 2016)

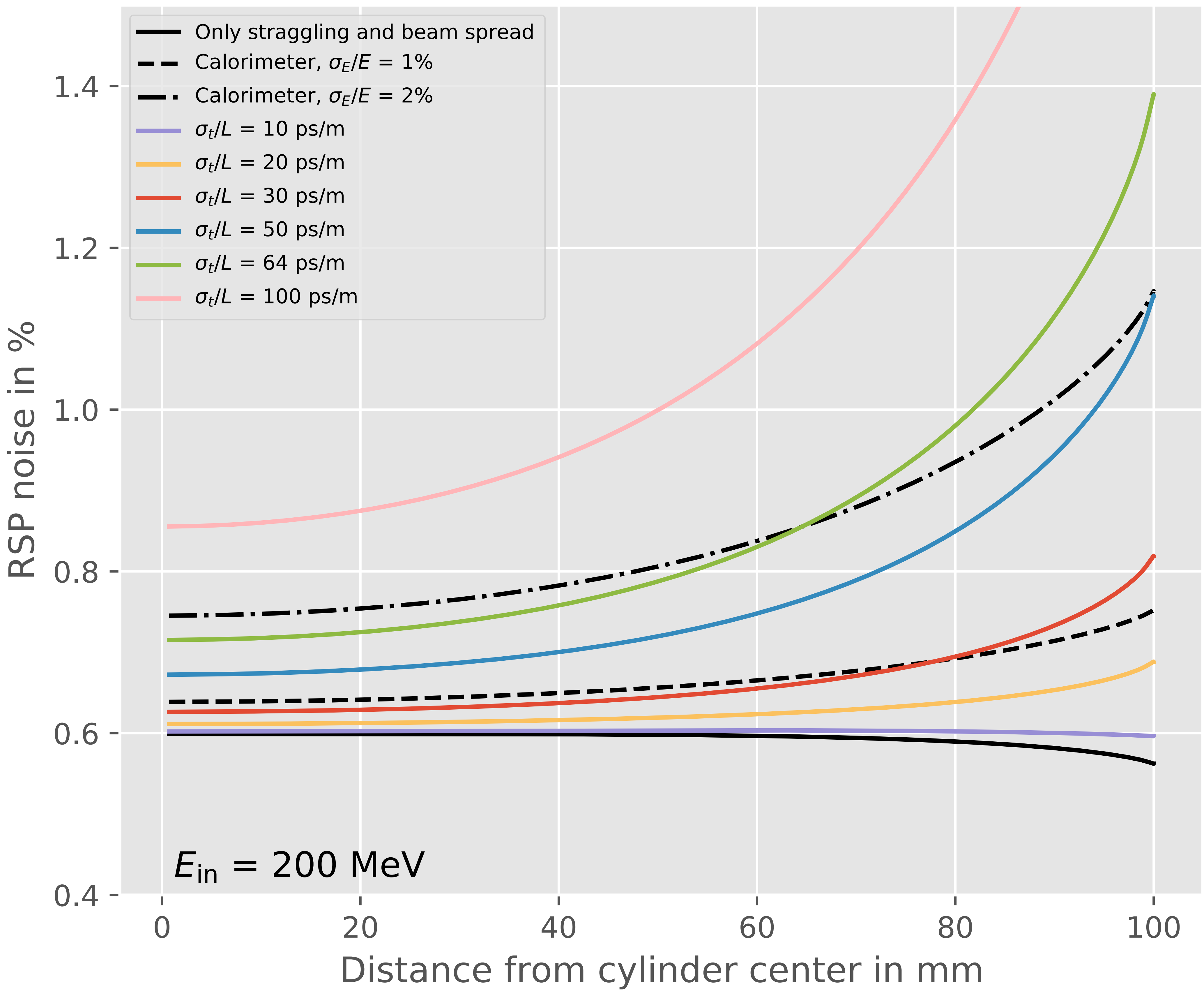
RSP uncertainty in a water cylinder

- Diameter: 20 cm
- Dose to center: 10 mGy (full acquisition)

Beam energy: 180 MeV

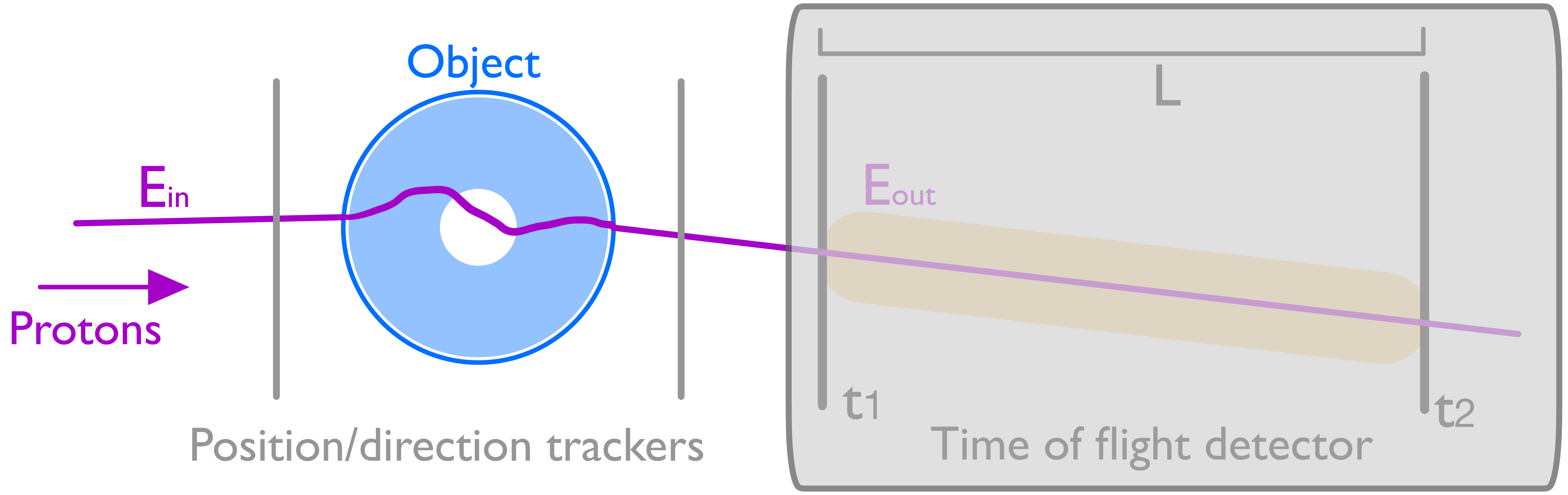


Beam energy: 200 MeV

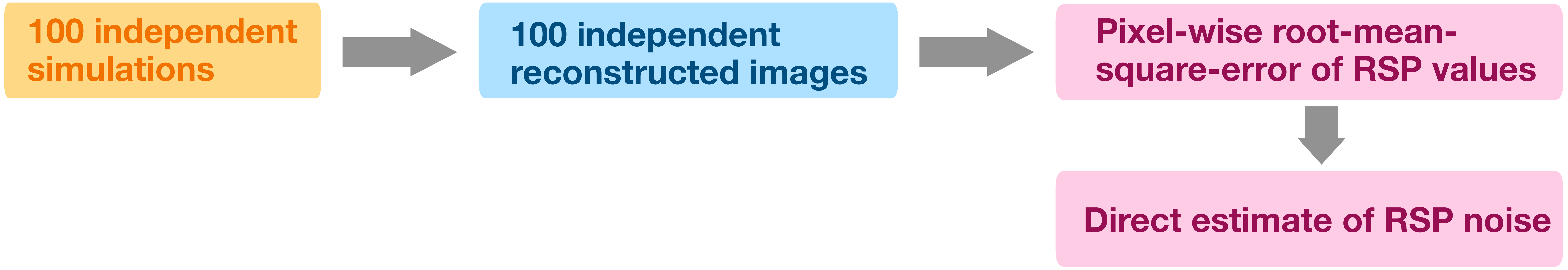


Monte Carlo simulations

- Geant4/GATE simulation
- Dose to center: 10 mGy (full acquisition)
- Phantom: water cylinder with 20 cm diameter
- QGSP_BIC physics list and ideal selection of protons which have only undergone electromagnetic interactions
- Ideal position and direction scoring



- Simulated as ideal energy detector
- Energy uncertainty added in post-processing



Noise due to multiple Coulomb scattering

- Multiple Coulomb scattering (MCS) deviates ions onto stochastic non-linear paths.
- Ions binned into the same pixel have traversed different phantom regions.
- This leads to WEPL variation if density gradients are present and near the object's edge [1,2].

Estimated MCS contribution from Monte Carlo results:

contains all noise contributions

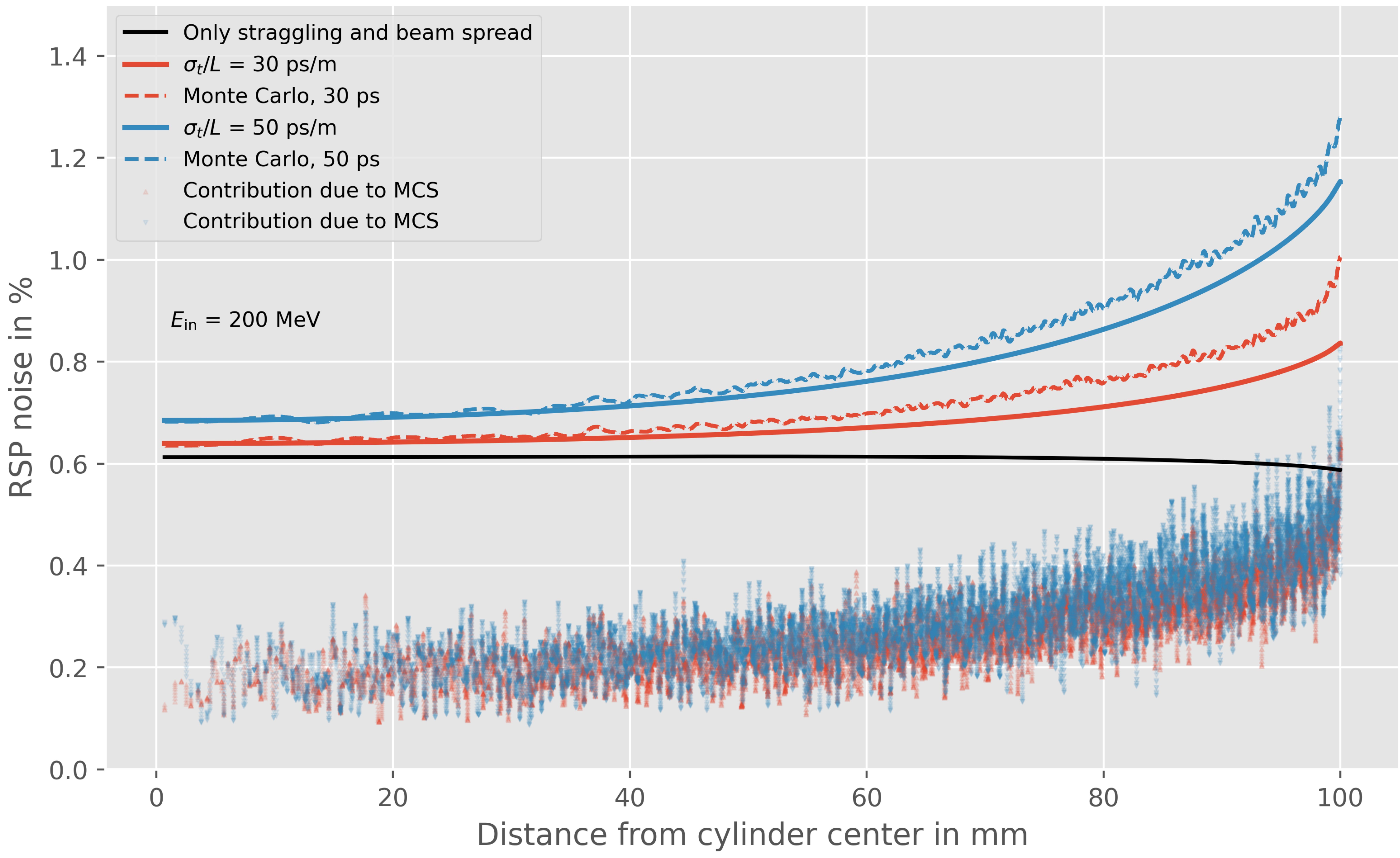
$$\mathrm{Var}_{\mathrm{RSP,MCS}} = \mathrm{Var}_{\mathrm{RSP,MC}} - \mathrm{Var}_{\mathrm{RSP,model}}$$

contains all noise contributions except for MCS

[1] Rädler, M. et al. (2018). Two-dimensional noise reconstruction in proton computed tomography using distance-driven filtered back-projection of simulated projections. Physics in Medicine & Biology, 63(21), 215009. https://doi.org/10.1088/1361-6560/aae5c9

[2] Dickmann, J., Wesp, P., Rädler, M., Rit, S., Pankuch, M., Johnson, R. P., … Dedes, G. (2019). Prediction of image noise contributions in proton computed tomography and comparison to measurements. Physics in Medicine & Biology, 64(14), 145016. https://doi.org/10.1088/1361-6560/ab2474

Monte Carlo results



Proton vs helium

Observations:

Helium mass = 4 x proton mass

Helium stopping power = 4 x proton stopping power

At equal residual range:
helium beam energy = 4 x proton beam energy

All energy and mass terms scale by factor of 4!

$$\sigma^2_{\mathrm{WEPL,He}} = \frac{1}{S^2_{\mathrm{w,p}}\Phi\Delta\xi^2}\left(\frac{1}{4}\sigma^2_{E_{\mathrm{out}},\mathrm{strag,p}} + \frac{\sigma^2_{t,\mathrm{He}}}{\sigma^2_{t,\mathrm{p}}}\sigma^2_{E_{\mathrm{out}},\mathrm{TOF,p}} + (\delta E_{\mathrm{beam,He}}E_{\mathrm{in,p}})^2\right)$$

Ratio of measurement errors:

$\frac{\sigma_{t,\mathrm{He}}}{\sigma_{t,\mathrm{p}}} \approx \frac{1}{4}$ because detector response scales with stopping power

Proton vs helium: at equal dose

$$\sigma^2_{\mathrm{WEPL,He}} \approx \frac{1}{S^2_{\mathrm{w,p}}\Phi_{\mathrm{helium}}\Delta\xi^2}\left(\frac{1}{4}\sigma^2_{E_{\mathrm{out}},\mathrm{strag,p}} + \frac{1}{4}\sigma^2_{E_{\mathrm{out}},\mathrm{TOF,p}} + (\delta E_{\mathrm{beam,He}}E_{\mathrm{in,p}})^2\right)$$

Dose scales with stopping power:

$D \propto S$ **and** $S_{\mathrm{helium}} \approx 4S_{\mathrm{proton}}$

Therefore, at equal dose:

$$\frac{1}{\Phi_{\mathrm{helium}}} \approx 4\frac{1}{\Phi_{\mathrm{proton}}}$$

$$\sigma^2_{\mathrm{WEPL,He}} = \frac{1}{S^2_{\mathrm{w,p}}\Phi_{\mathrm{proton}}\Delta\xi^2}\left(\sigma^2_{E_{\mathrm{out}},\mathrm{strag,p}} + \sigma^2_{E_{\mathrm{out}},\mathrm{TOF,p}} + 4(\delta E_{\mathrm{beam,He}}E_{\mathrm{in,p}})^2\right)$$

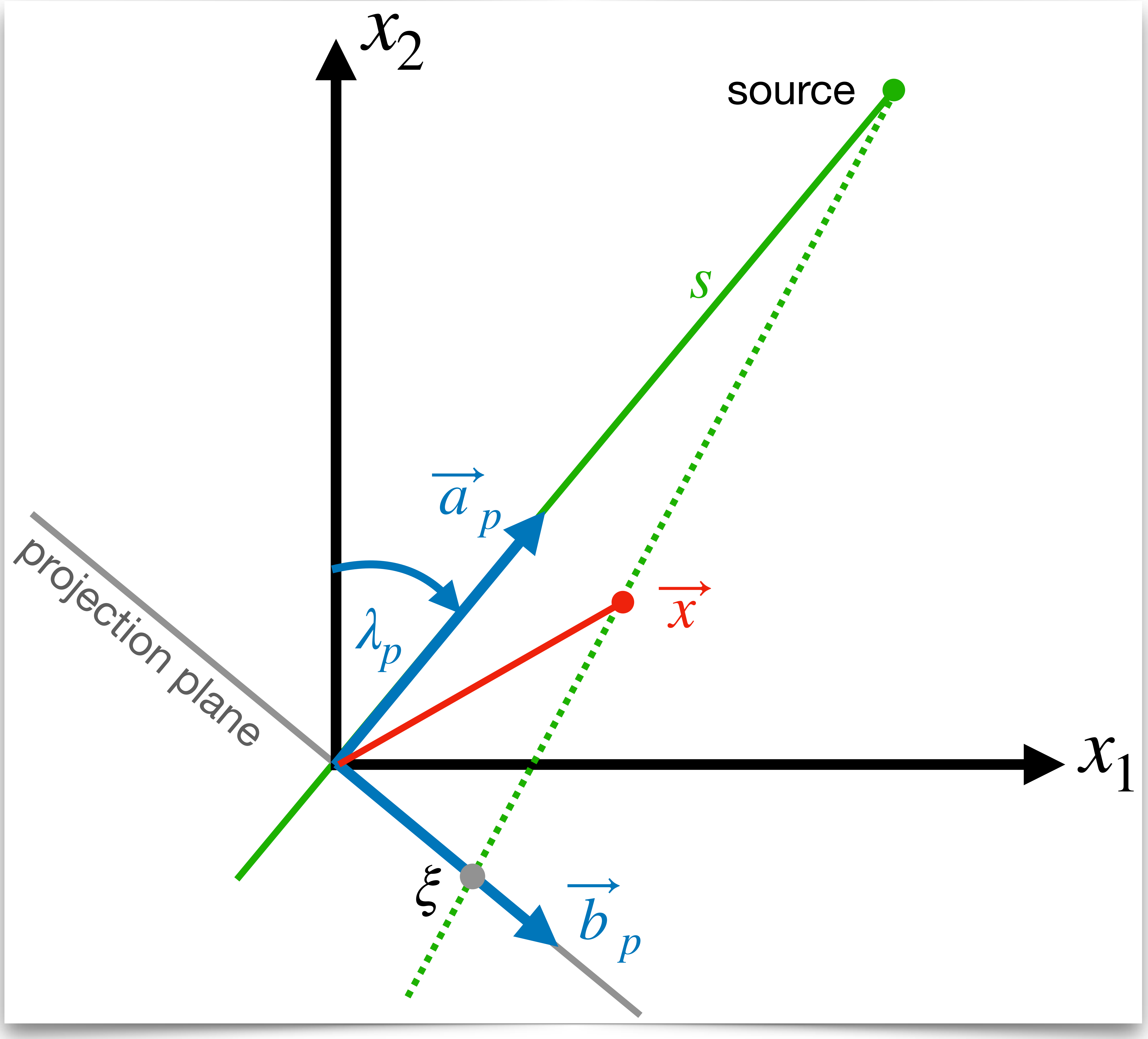
Protons and helium ions expected to yield similar noise.

Conclusion

- Time-of-flight is an alternative method for energy-loss measurement in ion CT
- RSP resolution better than 1% with velocity errors <50 ps/m
- At 30-50 ps/m velocity error: image noise is comparable with calorimeter-based system with 1-2% error
- Noise can be improved by optimizing incident beam energy as a function of expected water equivalent path length, e.g. via optimization similar to Dickmann et al. 2019
- Image noise expected to be similar with protons and helium ions.
- Interesting novel sensor technology from field of particle physics, e.g. LGAD (see talk by Albert)

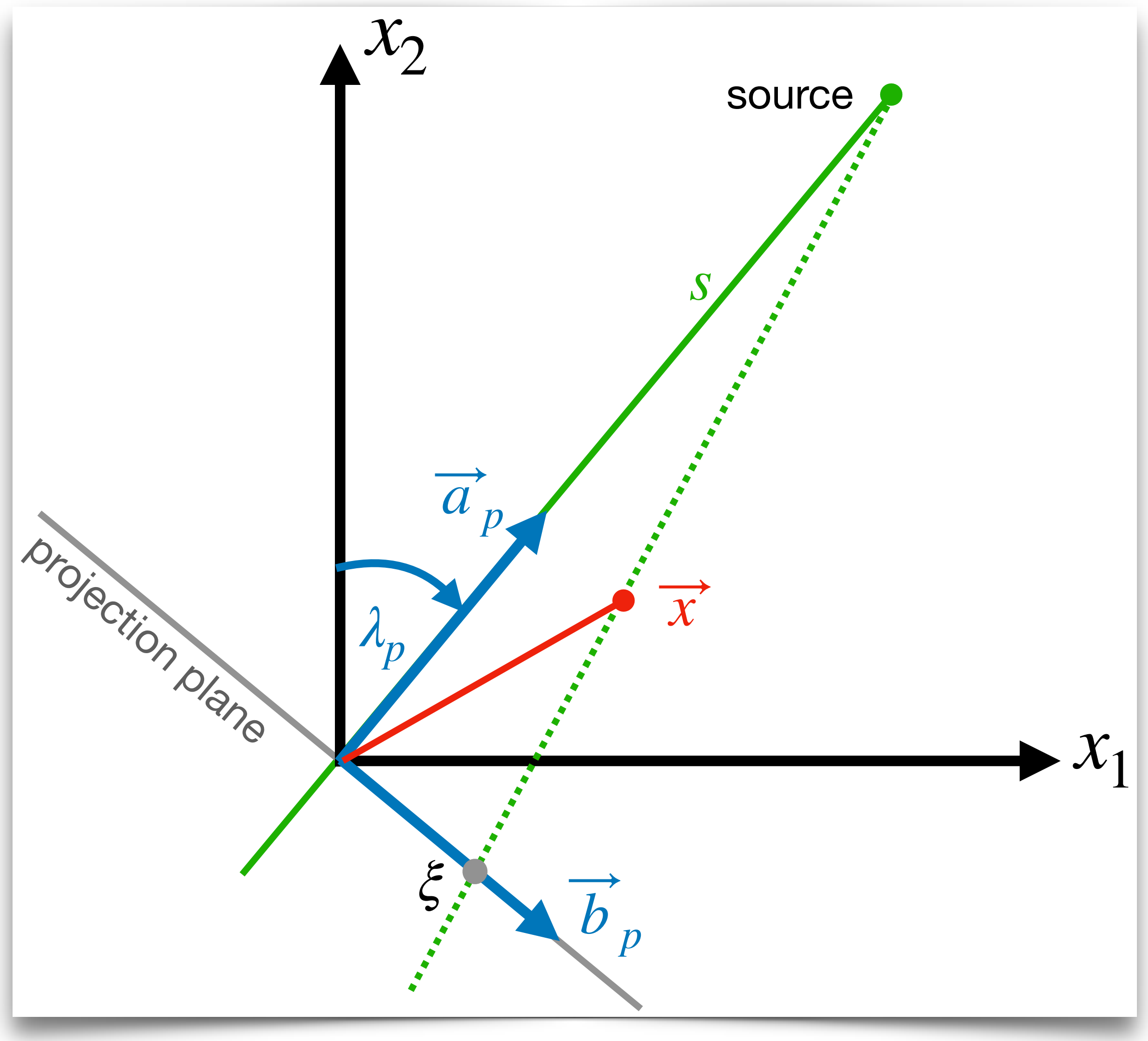
Thanks

Noise reconstruction: geometry (2D)



RSP uncertainty via noise reconstruction

(approximate)



Backprojection:

$$\mathrm{Var}_{\mathrm{RSP}}(\vec{x}) = f_{\mathrm{interp}} \frac{\Delta\lambda^2}{4} \sum_{p=1}^{P} \left(\frac{\|\vec{a}_{\lambda_p}\|}{\|\vec{x} \cdot (\vec{a}_{\lambda_p}/s) - \vec{a}_{\lambda_p}\|} \right)^4 V_p(\xi_k)$$

approximation

weighting factor

Filtering:

$$V_p(\xi_k) = (\Delta\xi)^2 \sum_{j=-J}^{J-1} h_F^2(\xi_k - \xi_j) \frac{\|\vec{a}_{\lambda_p}\|^2}{\|\vec{a}_{\lambda_p}\|^2 + \xi_j^2} \mathrm{Var}(\lambda_p, \xi_j)$$

weighting factor

WEPL variance in pixel j and projection p

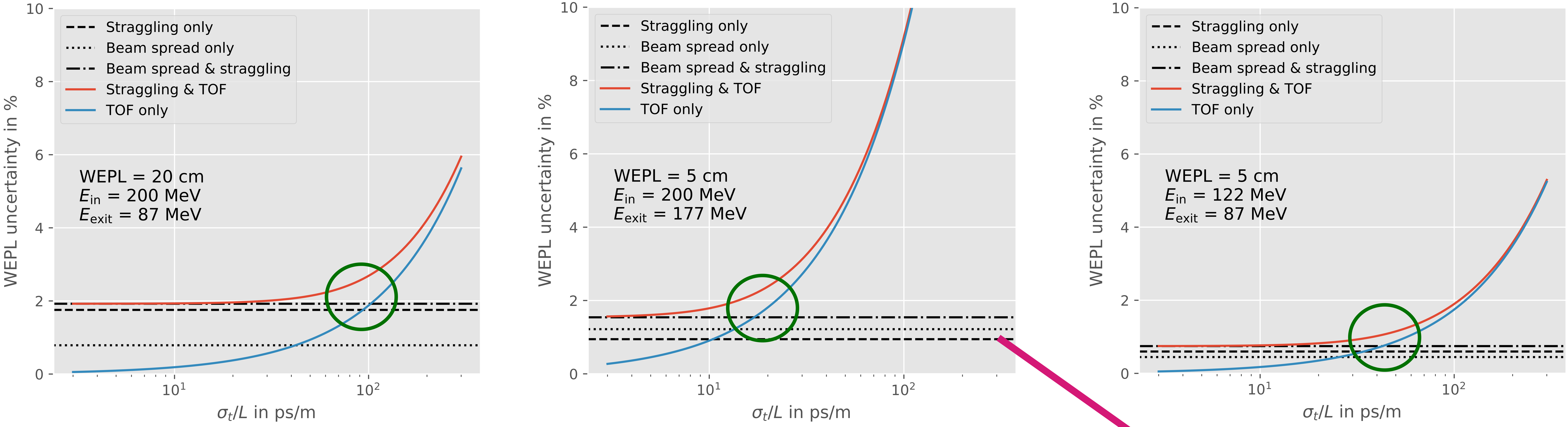
Anodized ramp filter:

$$h_F(\xi_j) = h_F((j + 1/2)\Delta\xi) = \begin{cases} 1/(2\Delta\xi)^2 & \text{for } j = 0, \\ 0 & \text{for } j \text{ even and } j \neq 0 \\ -1/(j\pi\Delta\xi)^2 & \text{for } j \text{ odd,} \end{cases}$$

[1] Wunderlich, A., & Noo, F. (2008). Image covariance and lesion detectability in direct fan-beam x-ray computed tomography. Physics in Medicine and Biology, 53(10), 2471–2493. https://doi.org/10.1088/0031-9155/53/10/002

WEPL uncertainty: TOF vs straggling

$$\sigma^2_{\mathrm{WEPL,N=1}}(E_{\mathrm{out}}) = \frac{\sigma^2_{\Delta E}(E_{\mathrm{out}})}{S^2_w(E_{\mathrm{out}})}$$



Intrinsic lower limit

Ideally: Incident energy should be adjusted as a function of (expected) WEPL (see Stefanie's talk yesterday)