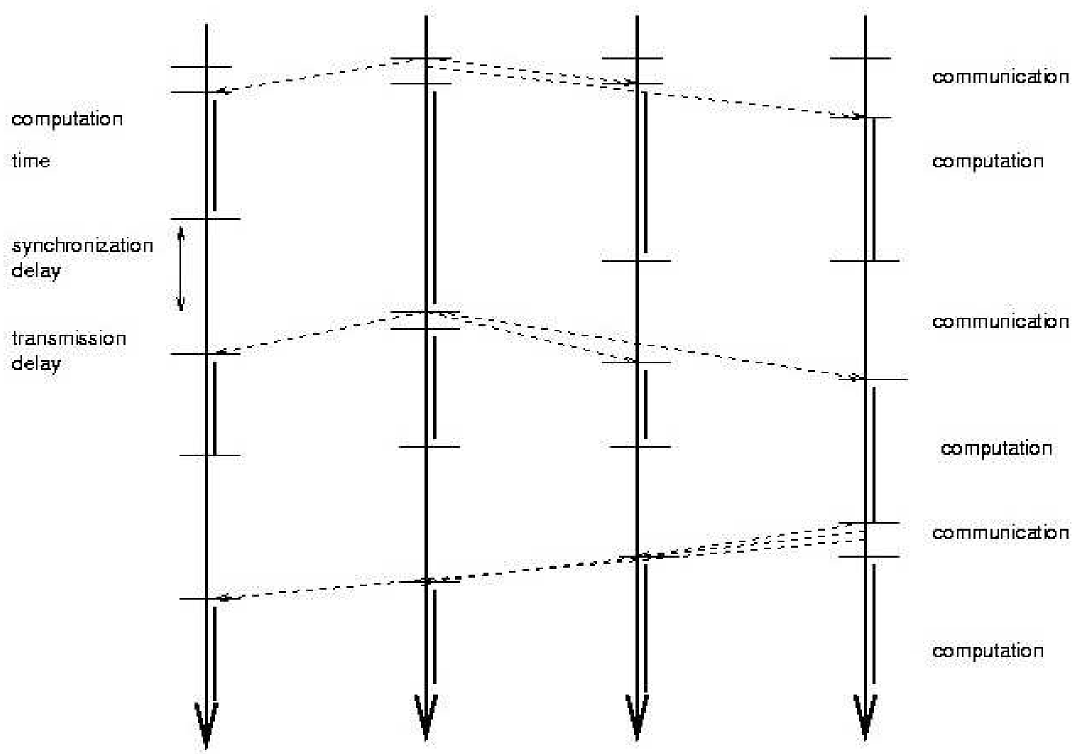
A model for Entropy of Parallel Execution

Ernesto Gomez
School of Engineering
and Computer Science
CSU San Bernardino
San Bernardino, CA

Keith E. Schubert
and Ritchie Cai
Department of Electrical and
Computer Engineering
Baylor University
Waco, TX 76798

5TH ANNUAL LLU ALGORITHMS IN PARTICLE IMAGING AND TREATMENT WORKSHOP

Irregular execution and communications



Parallel execution:

Definition 2.1. *A **parallel state** is a pair $S_{G,J} = (x_J, D_J^G)$, indexed by a step counter J which is a multi-index $J = J_G = \{(j_p) \mid p \in G\}$.*

Definition 2.2. *A **parallel transition** is a relation $S_{G,J} \to S_{H,K}$ between a pair of parallel states such that*

- $G \cap H \neq \emptyset$
- *for each $p \in G \cap H$, we have $k_p - j_p \leq 1$, and*
- *for at least one $p \in G \cap H$ we have $k_p - j_p > 0$.*

Definition 2.3. *Let $T = (S_{G^1,J^1} \to S_{G^2,J^2} \to \dots)$ be any sequence of parallel transitions relating parallel states. (Note that $G_i \cap G_{i+1}$ needs to contain at least one element, but there is no other restriction on the G_i). The p-**trace** of T is the list of serial state indices $\tau_p = (j_p^{i_1}, j_p^{i_2}, \dots, j_p^{i_k})$ in the order in which those states appear in T. Note that, since p is not necessarily found in every G_i, the superscripts i_ℓ are not necessarily consecutive. We will say that a p-trace τ_p is **time-like** if $j_p^{i_{\ell+1}} \geq j_p^{i_\ell}$ for all ℓ.*

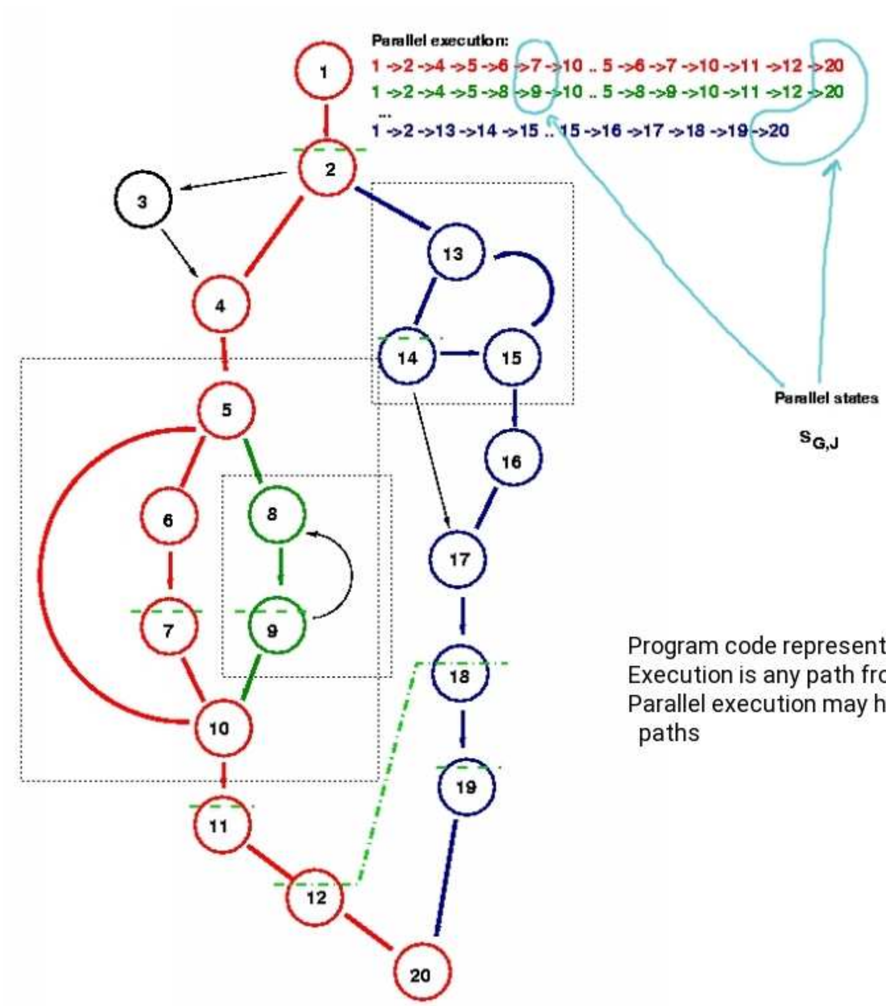
Definition 2.4. *An **SPMD parallel execution** E is a directed graph of transitions starting at $E_{G_{tot}} = S_{G_{tot},0}$ all of whose p-traces (def: 2.3) are time-like.*

Motivation:

1. Experimentally explore non-determinism in parallel state and execution

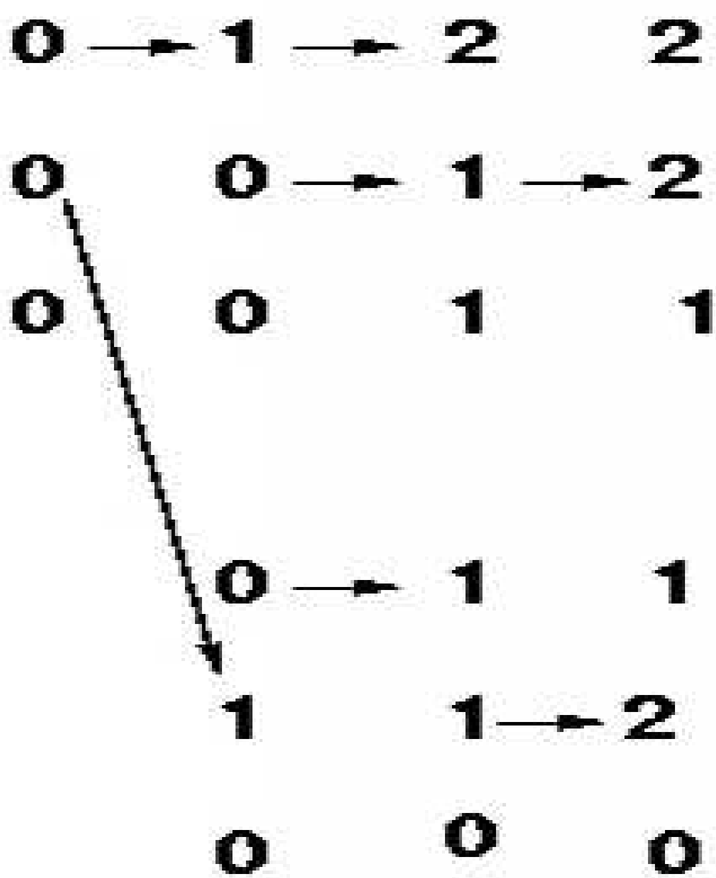
2. Verify that synchronization is detectable in execution, find other features that consistently appear in visualization

3. Characterize hardware-independent costs of synchronization (and other constructs) by analysis of entropy



Program code represented as basic blocks
Execution is any path from start to end block
Parallel execution may have different concurrent paths

non–deterministic transition



Parallel state is list of concurrent blocks
-> multiple possible successor states

SPMD model used for simpler representation, but not restricted to this model.

Represent code as Control Flow Graph CFG

Process q state is basic block x and memory contents M at step j

$$\sigma_{q,j} = (x_j, M_j)$$

Execution is a series of states representing a walk through CFG

$$S_{\Gamma,J} = \{\sigma_{i,j}\}$$

A parallel state is the set of sequential states ocurring concurrently

Gamma is the ordered set of processes and J the ordered set of step numbers - a parallel transition occurs when 1 or more processes advance - even if processes are deterministic, parallel transition is not

Representing each state as a P-tuple of concurrent block numbers gives us a P dimensional hypercube with side=number of blocks.

Some features in the phase space repeat (synchronization code in specific blocks). Most states are not deterministic and won't repeat.

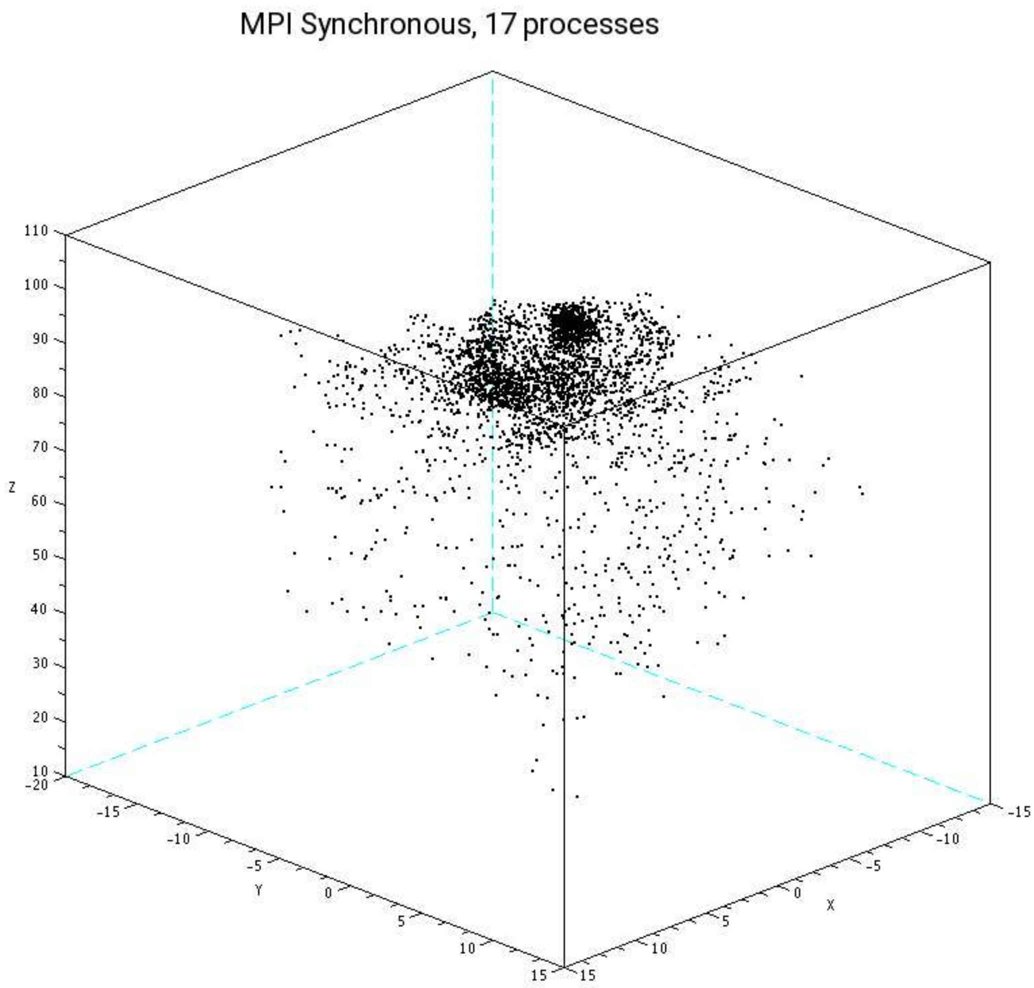
Given a set of points corressponding to one or a set of executions, Gibbs-Shannon entropy is:

$$-\sum p_i \log_2(p_i)$$

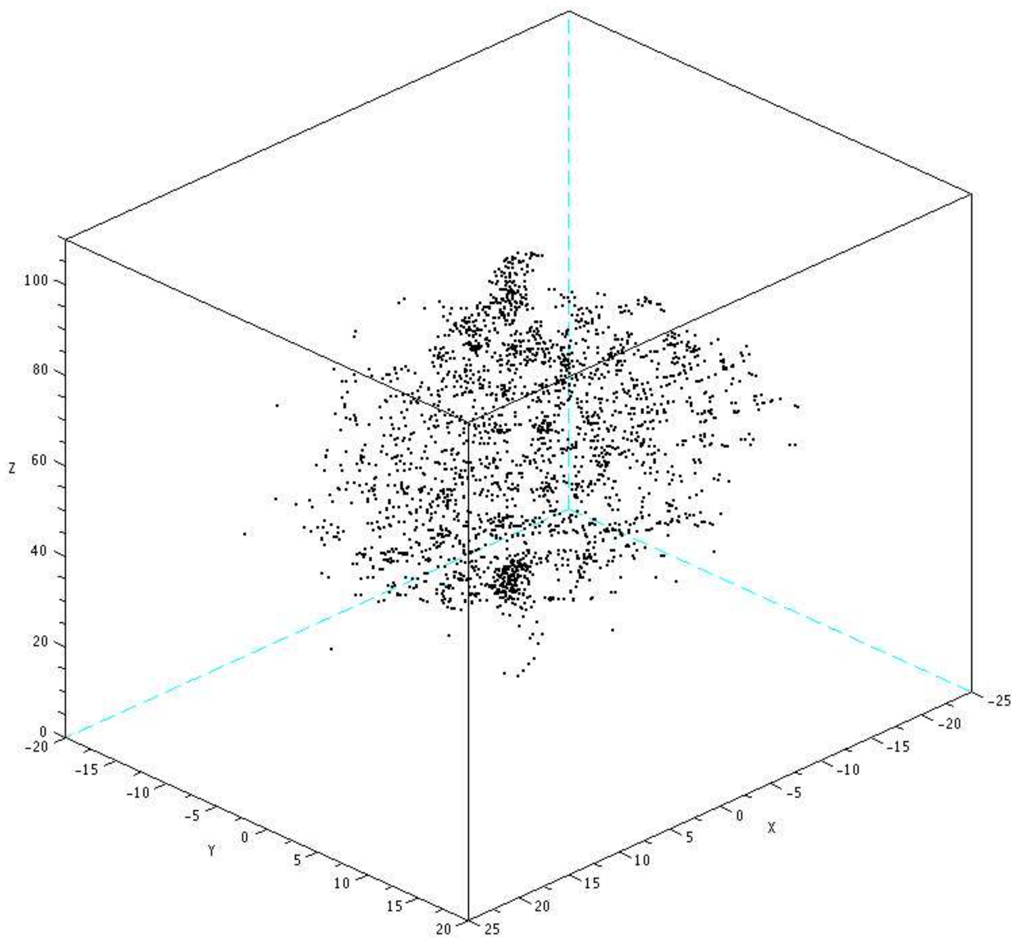
We use standard combinatoric properties to caculate the probabilities:

$$p_i = \frac{P!}{N^P \prod(b_k)!}$$

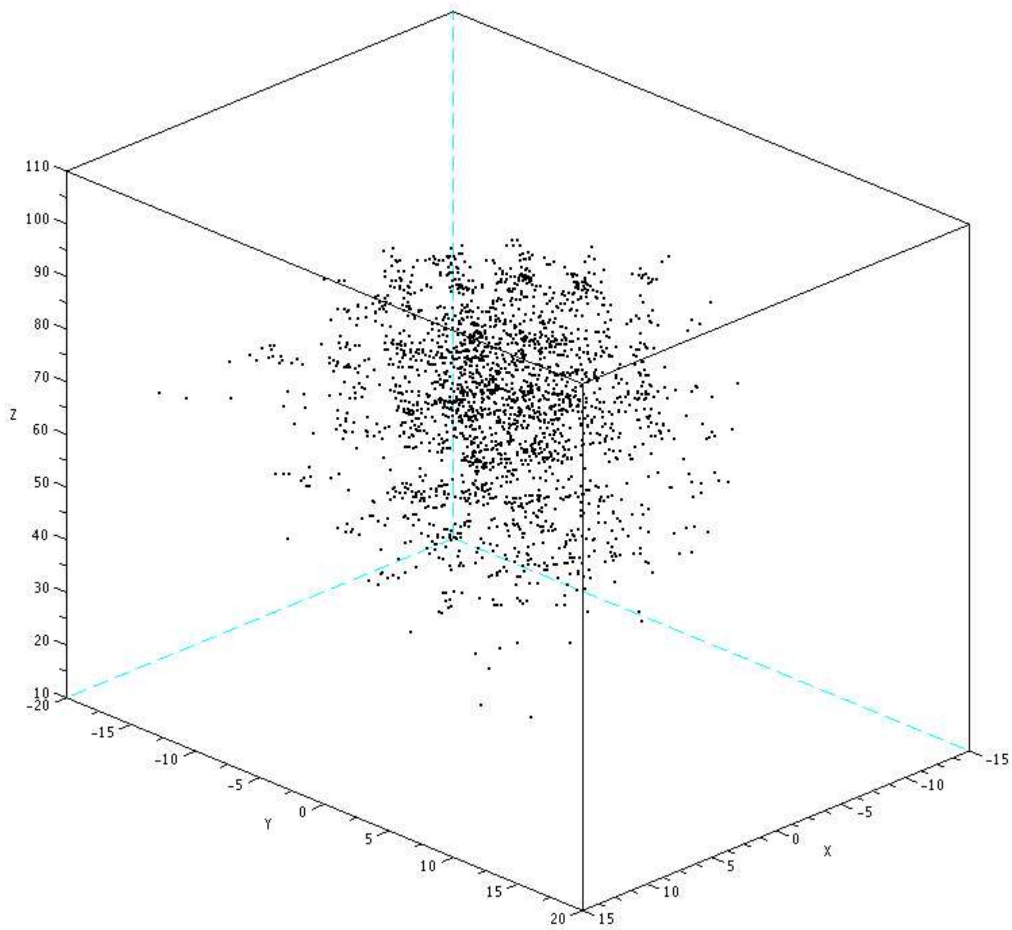
Where the b_k are subsets in the state with the same block number and we normalize to the total phase space hypervolume


MPI Synchronous, 17 processes
Z
110
100
90
80
70
60
50
40
30
20
10
-20
-15
-10
-5
Y
0
5
10
15
15
10
5
0
X
-5
-10
-15

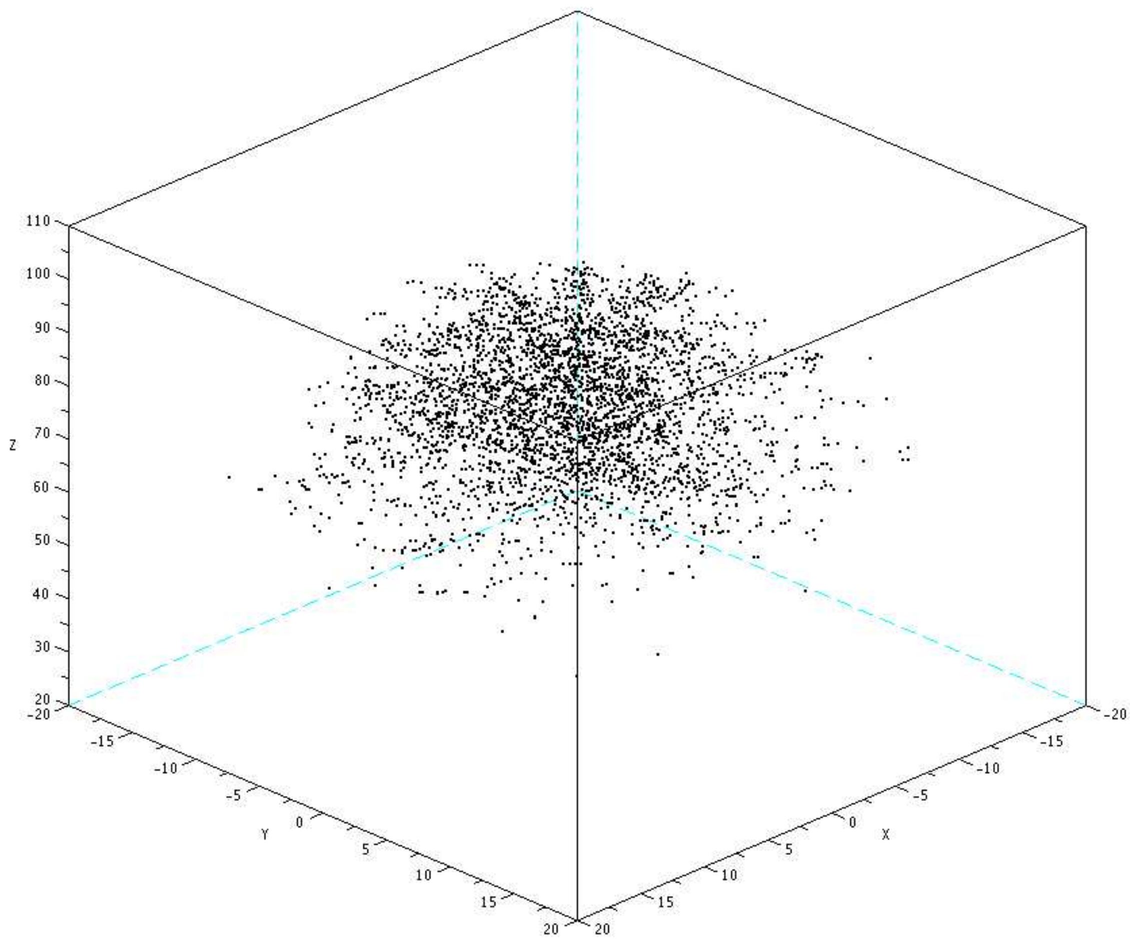
SOS asynchronous, 17 processes



No communication , 17 processes



No communications, guided, 17 processes

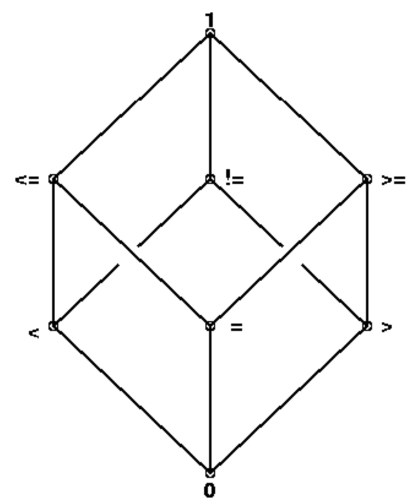


Algebra

S={1,≥,≤,=,>,<,0} is complete

Lattice closed under intersection, union and complement, with 0 and 1 is a Boolean Algebra

S with set operations is $\mathbf{B}^3$



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phase space reduction:
    ">" synchronization removes 1 quadrant - R=.75 of phase space per dimension
    "=" synchronization removes 2 quadrants - R=.5 of phase space per dimension

given unsynchronized entropy S, then entropy with synchronization S' is given by
        S' = log( exp(S)*R^N )) (given interval between synch longer than relaxation time)

Time prediction - dt/t = log(N)* dS/S (the log N factor assumes collective communication tree)
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AMOEBA - time:
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TARDIS: comparison of amoebampi and amoebaM (no synch)

17 amoebasos.mat entropy = 12.06215727 mean state size 5.340149838 time 2550742
17 amoebampi.mat entropy = 11.40358976 mean state size 5.660535992 time 1842581
17 amoebaN.mat entropy = 12.53261805 mean state size 5.178458642 time 991982
17 amoebaM.mat entropy = 14.42974945 mean state size 4.930923558 time 603365

dt/T = 0.763455104436278 (expected)
dt/T = 0.46486557378577 (actual


RAVEN: comparison of amoebampi and amoebaM (no synch)

17 amoebasos.mat entropy = 11.79212385 mean state size 5.21425634 time 13013195
17 amoebampi.mat entropy = 10.85077704 mean state size 5.616563899 time 6372684
17 amoebaN.mat entropy = 12.78105428 mean state size 4.997991238 time 3577237
17 amoebaM.mat entropy = 16.32720129 mean state size 4.387070064 time 981748

dt/T = 0.924557497217248 (expected)
dt/T = 0.78695924005089



AMOEBA - entropy prediction
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RAVEN: 11.4094047683197 - expected 11.79

TARDIS: 9.40940476831973  - expected 12.06215727
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